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FRANCISCO BLASQUES

FALK BRÄUNING

IMAN VAN LELYVELD

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Francisco Blasques[a,b,c] and Falk Bräuning[a,c] and Iman van Lelyveld[d]

[(a)]Department of Econometrics, VU University Amsterdam

[(b)]Department of Finance, VU University Amsterdam

[(c)]Tinbergen Institute [(d)]De Nederlandsche Bank

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Abstract

This paper introduces a structural micro-founded dynamic stochastic network model for the unsecured interbank lending market. Banks are profit optimizing agents subject to random liquidity shocks and can engage in costly counterparty search to find suitable trading partners and peer monitoring to reduce counterparty risk uncertainty. The structural parameters are estimated by indirect inference using appropriate network statistics of the Dutch interbank market. The estimated model is shown to explain accurately important dynamic features of the interbank market network. In particular, monitoring of counterparty risk and directed search are shown to be key factors in the formation of interbank trading relationships that are associated with improved credit conditions. Finally, the model is used to filter the optimal search and monitoring expenditures in the network and to analyze optimal network responses to changes in the policy of the central bank.

1 Introduction

The global financial crisis of 2007-2008 highlighted again the crucial role of interbank lending markets for the financial system and the entire economy. Particularly, after the September 2008 fall of Lehman, dramatic increases in perceived counterparty risk led to severe distress of unsecured interbank markets. As a result, monetary policy implementation was hampered and credit supply to the non-financial sector dropped substantially with adverse consequences for both the financial sector and the real economy. Moreover, the freeze of the Euro area interbank markets within some member countries severely

*Corresponding author: `F.Brauning@vu.nl`. Blasques and Bräuning thank the SWIFT institute for financial support. The opinions expressed in this paper do not necessarily reflect those of De Nederlandsche Bank or the Eurosystem.

amplified the sovereign debt crisis in Europe. In order to avoid these effects, central banks intervened not only by injecting additional liquidity in the banking sector but also by adjusting their monetary policy instruments. As a consequence, central banks became the intermediary for large parts of the money market during the crisis.

This experience further stimulated the debate about the organizational structure of interbank markets, in particular if the current decentralized market should be replaced by a system with a central counterparty. Generally a key benefit of having a central counterparty to the unsecured interbank market is that it reduces systemic risk as credit exposures between banks can no longer give rise to a chain reaction that might bring down large parts of the banking sector, see Allen and Gale (2000). Likewise search frictions that result from asymmetric information about liquidity conditions of other banks are mitigated. On the other hand, with a central counterpart, private information that banks have about the credit risk of other banks is no longer reflected in the price at which banks can obtain liquidity. Thus market discipline is impaired. Moreover, the incentives for banks to acquire and process such information are largely eliminated. However, as Rochet and Tirole (1996) argue, if banks can assess the creditworthiness of other institutions more efficiently than a regulatory authority, a decentralized organization of the interbank market can be optimal.[1] Consequently, in order to assess whether a central counterpart in the unsecured money market is welfare enhancing one has to gauge the extent to which private information about counterparty credit risk and search frictions affect the liquidity allocation in money markets.

Our paper contributes to this debate by introducing and estimating a micro-founded structural dynamic stochastic network model to analyze observed cross-sectional and inter-temporal variation in interbank credit availability and conditions. The key economic concept underlying the model is that of asymmetric information about counterparty risk and liquidity conditions elsewhere in the market. We focus on the role of peer monitoring and counterparty search targeted at mitigating this information asymmetry and the related search frictions in a decentralized market. We characterize the bilateral

[1] Indeed, also the ECB highlights the role of monitoring and private information: "Specifically, in the unsecured money markets, where loans are uncollateralised, interbank lenders are directly exposed to losses if the interbank loan is not repaid. This gives lenders incentives to collect information about borrowers and to monitor them over the lifetime of the interbank loan [...]. Therefore, unsecured money markets play a key peer monitoring role.", see the speech by Benoît Cœuré, Member of the Executive Board of the ECB, at the Morgan Stanley 16th Annual Global Investment seminar, Tourrettes, Provence, 16 June 2012. http://www.ecb.europa.eu/press/key/date/2012/html/sp120616.en.html, retrieved 10/10/2013

equilibrium interest rates as an increasing function of the outside option for borrowing (given by the central bank's standing facilities), counterparty risk uncertainty and the lender's market power relative to the borrower. Further we show that the optimal search and monitoring strategy towards all other banks in the network are functions of the expected profitability of trade with each counterparty. Specifically, we find that the optimal dynamic monitoring decision depends on the expected probability of being contacted by distinct borrowers in the future and the expected loan volumes that can be realized with these borrowers. Moreover, optimal counterparty search depends on expected volumes and offer rates in each period.

The second novelty of this paper lies in the econometric analysis of the model, in particular the estimation of the structural parameters. In this respect, we go beyond recent interbank research that has focused on theoretical modeling only and did not tackle parameter estimation; see for instance Heider et al. (2009) and Afonso and Lagos (2012), and for more general trade network models Weisbuch et al. (2000) and Babus (2011). However, the increased availability of granular interbank lending data resulting from payment records makes it appealing to develop a structural economic model that can be estimated from these data sets. Specifically, we make use of a unique panel of unsecured overnight loans between the largest 50 Dutch banks derived from the Target2 payment system records from 2008 to 2011. Given the dynamic complexity of the model and the presence of latent variables, estimation with classical estimators is largely impossible and we turn to the unifying approach of indirect inference for simulation-based parameter estimation, introduced in Gourieroux et al. (1993). Indirect inference of the structural parameters is based here on a number of descriptive network statistics, e.g. density, reciprocity, clustering and centrality, that became popular in characterizing the topological structure of interbank markets, see Bech and Atalay (2009) for the US, van Lelyveld and in 't Veld (2012) for the Netherlands and Abbassi et al. (2013) for the Euro area. We further complement these network statistics by moment statistics of the data and bilateral lending relationship measures as in Furfine (1999) and Cocco et al. (2009).

The estimated model is shown to accurately explain several crucial features of the Dutch interbank market. In particular, the estimated structural parameters reveal that search frictions, counterparty risk uncertainty and peer monitoring are significant factors in matching the dynamic structure of the data in particular the high sparsity and stability of the network. As a result of the interrelation between monitoring and search, some banks form long-term trading relationships that are associated with lower interest rates and improved credit availability. In particular, peer monitoring and search efforts crucially depend on banks' heterogeneous and persistent expectations about credit availability and conditions. Our findings also highlight the role of heterogeneous liquidity shock distributions across banks for the interbank network structure and distribution of loan volumes. The model is then used to filter the optimal search and monitoring expenditures in the market taking into account the uncertainty of the true network structure. Furthermore, we analyze responses of the interbank lending network to changes in the ECB interest rate corridor. We show that by increasing the corridor width the ECB fosters interbank market lending via directly altering the outside options and increasing the potential surplus of interbank lending. However, we document a further indirect multiplier effect as increased expected surplus from interbank trading will eventually intensify banks' monitoring and search that in turn further improve credit conditions and availability in the market leading to yet more liquidity.

This paper is closely related to recent work by Afonso and Lagos (2012) who also resort to a dynamic stochastic modeling framework to study the distribution of interest rates and volumes. However, these authors focus on the role of search frictions for intra-day trading dynamics in the fed funds market assuming no counterparty risk. On the other hand, building on the classical banking model by Diamond and Dybvig (1983), Freixas and Holthausen (2005) and Heider et al. (2009) have focused on asymmetric information about counterparty risk in competitive and anonymous markets, thereby abstracting from the decentralized network structure where deals are negotiated on a bilateral basis and credit conditions crucially depend on heterogeneous expectations about counterparty risk and credit conditions. The proposed model thus provides a unified description of interbank lending that takes into account these two key frictions. This paper is also related to empirical studies based on reduced form regressions by Furfine (1999), Furfine (2001), Ashcraft and Duffie (2007), Cocco et al. (2009), Craig and von Peter (2010), Bräuning and Fecht (2012) and Afonso et al. (2013) that showed that the network structure and banking relationships crucially affect interbank credit conditions. In our model repeated interaction with the same counterparties arises endogenously as banks may target peer monitoring and counterparty search towards preferred counterparts and we thereby provide a formal theory of interbank relationship lending and network formation. In the latter respect this research also relates to theoretical work on the formation of financial networks, see for instance Gale and Kariv (2007), Babus (2011) and Hommes et al. (2013).

The paper is structured as follows. Section 2 introduces the structural model, defines structural forms, the agents' expectation formation mechanisms and finally obtains a reduced form representation of the structural form. Section 3 provides details on the estimation procedure, discusses the parameter estimates of the model and analyzes their relative fit in terms of various criteria. Finally, section 4 analyzes the estimated model and studies policy implications. Section 5 concludes.

2 The Interbank Network Model

The fundamental driving force of interbank lending are the liquidity shocks that hit the banks' payment accounts in their daily business operations. Banks wish to smooth these shocks by using the central bank's standing facilities or by borrowing and lending funds from each other in a decentralized unsecured interbank market. We model this market as a network consisting of N nodes with a time-varying number of directed links between them. Each node represents a bank and each link represents an interbank loan. Time periods are indexed by $t \in \mathbb{N}$. Banks in the interbank market are indexed by $i \in \{1, ..., N\}$.

At every period, banks are subject to liquidity shocks and enter the market with the objective to maximize expected discounted interbank market profits by: *(i)* choosing which banks to approach for transaction; *(ii)* bilaterally bargaining interest rates with other banks in the interbank market that are available for trade; and *(iii)* selecting the amount of monitoring expenditures directed at gaining knowledge about future probabilities of default.

We make the problem operational by specifying a probabilistic structure for the liquidity shocks faced by banks, modeling the bilateral bargaining problem that banks face and the allocation of resources to peer monitoring and counterparty search.

2.1 Liquidity Shocks

At each period t, every bank enters the market as a potential borrower or lender according to the random vector $\zeta_t = (\zeta_{1,t}, ..., \zeta_{N,t})'$. Each element $\zeta_{i,t} \in \mathbb{R}$, denotes the period t liquidity shock of bank i and we interpret $\zeta_{i,t} < 0$ as a deficit and $\zeta_{i,t} > 0$ as a surplus.

A liquidity deficit might result, for instance, from customers withdrawing deposits from their bank accounts, or because the bank has an investment opportunity and needs to collect funds for this reason.

The distribution of liquidity shocks in the banking system has been identified as one key driver of interbank lending by Allen and Gale (2000) amongst others. We assume that liquidity shocks $\zeta_{i,t}$ are normally distributed with bank specific mean μ_{ζ_i} and variance $\sigma^2_{\zeta_i}$ parameters

$$\zeta_{i,t} \sim \mathcal{N}(\mu_{\zeta_i}, \sigma^2_{\zeta_i}) \quad \text{where} \quad \mu_{\zeta_i} \sim \mathcal{N}(\mu_\mu, \sigma^2_\mu) \quad \text{and} \quad \log \sigma_{\zeta_i} \sim \mathcal{N}(\mu_\sigma, \sigma^2_\sigma).$$

We assume (conditional) independence and normality of liquidity shocks for convenience as it allows to analytically compute part of the model's solution. The heterogeneity in the liquidity shocks across banks allows us to model size effects related to the scale of a bank's business through larger variances that are drawn from a log-normal distribution. Further this heterogeneity allows us to account for structural liquidity provision or demand by some banks through a nonzero mean $\mu_{\zeta_{i,t}}$. Indeed empirical research has shown that large banks trade much more often than small banks and at higher loan volumes, and that banks that specialize in collecting deposits typically provide liquidity to the interbank market, see Furfine (1999), Bräuning and Fecht (2012), and also the core-periphery analysis of interbank markets initiated by Craig and von Peter (2010).

2.2 Counterparty Risk Uncertainty

Due to the unsecured nature of interbank lending a borrower may default on an interbank loan and impose losses on its lenders. The *true probability of default* of bank j at time t is denoted $P_{j,t}$ and obtained as the tail probability of a random variable $z_{j,t}$ that measures the *true financial distress* of bank j. In particular, $z_{j,t}$ is constructed so that bank j is forced into default whenever $z_{j,t}$ takes values above some time-varying threshold $\epsilon_{j,t}$,[2]

$$P_{j,t} = \mathbb{P}(z_{j,t} > \epsilon_{j,t}).$$

[2]Note that the time-variation of the threshold is designed to capture changes over time in the distribution of z_t. Indeed, it would be equivalent to have a fixed threshold ϵ but a time-varying distribution for z_t so that $P_{j,t} = \mathbb{P}_t(z_{j,t} > \epsilon_t)$ with time-varying $\mathbb{P}_t$. The time-varying ϵ_t can thus be seen as a normalization of $\mathbb{P}_t$.

While counterparty credit risk relates to the riskiness and liquidity of a borrower's assets, asymmetric information about counterparty risk are seen as a major characteristic of the financial crisis that led to inefficient allocations in money markets, see Heider et al. (2009) for a theroretical model and Afonso et al. (2011) for empirical evidence.[3]

Asymmetric information problems arise because counterparty risk assessment is based on the *perceived probability of default* that bank i attributes to bank j at time t. This probability is denoted $P_{i,j,t}$ and obtained as the tail probability of a random variable $z_{i,j,t}$ that measures bank i's *perceived financial distress* of bank j. The perceived financial distress $z_{i,j,t}$ contains an added uncertainty that is modeled by the addition of an independent *perception error* $e_{i,j,t}$ so that,

$$z_{i,j,t} = z_{j,t} + e_{i,j,t}.$$

Note that if the sequence of perception errors $\{e_{i,j,t}\}_{t \in \mathbb{N}}$ is *iid* with $\mathbb{E}(e_{i,j,t}) = 0$ and small variance $\text{Var}(e_{i,j,t}) \approx 0$, then the perceived financial distress is correct on average $\mathbb{E}(z_{i,j,t}) = \mathbb{E}(z_{j,t})$ and added uncertainty is small $\text{Var}(z_{i,j,t}) = \text{Var}(z_{j,t}) + \text{Var}(e_{i,j,t}) \approx \text{Var}(z_{j,t})$. As a result the perceived probability of default $P_{i,j,t}$ is likely to follow closely the true probability of default $P_{j,t}$. If on the other hand, $\{e_{i,j,t}\}_{t \in \mathbb{N}}$ contains dynamics and its mean is allowed to deviate considerably from zero, or its variance is allowed to become large, then the perceived financial distress $z_{i,j,t}$ can be quite different from the true financial distress $z_{j,t}$ and the perceived probability of default $P_{i,j,t}$ might deviate considerably from the real probability of default $P_{j,t}$.

The evolution of the perception errors over time $\{e_{i,j,t}\}_{t \in \mathbb{N}}$, such as their mean and variance, will be determined by the knowledge that bank i has about the default risk of bank j. This knowledge will depend on factors such as the past trading history and the monitoring expenditure that bank i allocates to learn about bank j's financial situation, as discussed in more detail in the following section. We assume that perception errors have mean zero and variance $\tilde{\sigma}^2_{i,j,t} = \text{Var}(e_{i,j,t})$ that evolves over time according to

[3] In this respect William Dudley, President and CEO of the Federal Reserve Bank of New York, remarked (citation taken from Heider et al. 2011): «So what happens in a financial crisis? First, the probability distribution representing a creditor's assessment of the value of a financial firm shifts to the left as the financial environment deteriorates [...]. Second, and even more importantly, the dispersion of the probability distribution widens - lenders become more uncertain about the value of the firm. [...] A lack of transparency in the underlying assets will exacerbate this increase in dispersion. ("More Lessons from the Crisis", Nov. 13, 2009)»

autoregressive dynamics given by

$$\tilde{\sigma}^2_{i,j,t} = \xi(\phi_{i,j,t-1}, \tilde{\sigma}^2_{i,j,t-1}) + \delta_\sigma u_{i,j,t} = \alpha_\sigma + \frac{\gamma_\sigma}{1+\exp(\beta_\sigma \phi_{i,j,t-1})}\tilde{\sigma}^2_{i,j,t-1} + \delta_\sigma u_{i,j,t}. \quad (1)$$

Here $\phi_{i,j,t-1}$ is a function of past bilateral trade intensity and monitoring expenditure that measures the amount of *new information* that bank i collected about the financial situation of bank j in period $t-1$, and $u_{i,j,t} \sim \chi^2(1)$ is a shock to the counterparty risk uncertainty scaled by $\delta_\sigma > 0$. Moreover we assume that $\gamma_\sigma > 0$ and $\beta_\sigma > 0$. Hence the added information reduces the perception error variance. In particular, $\xi(\phi_{i,j,t-1}, \tilde{\sigma}^2_{i,j,t-1}) < \tilde{\sigma}^2_{i,j,t-1}$ for large enough $\phi_{i,j,t-1}$ thus allowing the perceived error variance to decrease $\tilde{\sigma}^2_{i,j,t} < \tilde{\sigma}^2_{i,j,t-1}$ with positive probability. Moreover, $\frac{\partial^2 \xi(\phi_{i,j,t-1}, \tilde{\sigma}^2_{i,j,t-1})}{\partial \phi^2_{i,j,t-1}} > 0$ and hence the effect of additional information is weaker for higher levels of $\phi_{i,j,t-1}$. This dictates decreasing returns to scale in information gathering.

Since the exact distribution of $z_{i,j,t}$ is unknown to bank i, every bank is assumed to approximate the tail probability of the extreme event of default by the conservative bound provided by Chebyshev's one-tailed inequality,[4]

$$\mathbb{P}(z_{i,j,t} > \epsilon_{i,j,t}) \leq \frac{\sigma^2_{i,j,t}}{\sigma^2_{i,j,t} + \epsilon^2_{i,j,t}} = \frac{\sigma^2_{j,t} + \tilde{\sigma}^2_{i,j,t}}{\sigma^2_{j,t} + \tilde{\sigma}^2_{i,j,t} + \epsilon^2_{i,j,t}} = P_{i,j,t},$$

with $\epsilon_{i,j,t} > 0$. Hence, both the true riskiness of a bank as well as the additional uncertainty resulting from the perception error increase the perceived probability of default on which lender banks base their credit risk assessment. Conditional on these bank-specific perceived probabilities of defaults banks bargain about the loan conditions.

2.3 Bargaining and Equilibrium Interest Rates

Without loss of generality, let bank i be the lender bank ($\zeta_{i,t} > 0$) in the bargaining process and bank j be the borrower bank ($\zeta_{j,t} < 0$). From the point of view of bank i lending funds to bank j at time t is a risky investment with stochastic return

$$R_{i,j,t} = \begin{cases} r_{ijt} & \text{w.p. } 1 - P_{i,j,t} \\ -\delta & \text{w.p. } P_{i,j,t}. \end{cases}$$

[4] Instead of the Chebyshev bound one can assume that the banks use a certain distribution to compute this probability. Then we just have to use the respective CDF here.

where we assume that loss given default is $\delta \times 100$ percent.[5] We further assume that bank i is risk neutral and maximizes expected lending profit conditional on $\sigma^2_{i,j,t}$ and $\epsilon_{i,j,t}$. The expected profit per dollar at an equilibrium interest rate $r_{i,j,t}$ is then given by

$$\mathbb{E}_t R_{i,j,t} = (1 - P_{i,j,t}) r_{i,j,t} - \delta P_{i,j,t}.$$

For the borrower bank j, the borrowing cost per dollar is simply given by the equilibrium interest rate $r_{i,j,t}$ when borrowing from lender bank i.

We follow the standard approach in search models and assume that banks negotiate interest rates bilaterally according to a generalized Nash bargaining, see Afonso and Lagos (2012) for a similar application to interbank markets. Written in terms of excess return relative to the outside options the bilateral equilibrium interest rate then satisfies

$$r_{i,j,t} \in \arg\max_r \left((1 - P_{i,j,t}) r - \delta P_{i,j,t} - \underline{r}_t\right)^{\theta_{i,j,t}} \left(\overline{r}_t - r\right)^{1-\theta_{i,j,t}},$$

where $\underline{r}_t$ is the outside option for lenders (e.g., the interest rate for depositing at the central bank), $\overline{r}_t$ is the the outside option for borrowers (e.g., interest rate for borrowing from the central bank) with $\overline{r}_t \geq \underline{r}_t$. The parameter $\theta_{i,j,t} \in [0, 1]$ denotes the bargaining power of lender i relative to borrower j. Clearly if $\theta_{i,j,t} = 1$ the lender is able to extract all rents from the borrower.

Normalizing $\underline{r}_t = 0$ and denoting $r_t = \overline{r}_t$, the corresponding equilibrium interest rate between lender i and borrower j is given by

$$r_{i,j,t} = \theta_{i,j,t} r_t + (1 - \theta_{i,j,t}) \frac{\delta P_{i,j,t}}{1 - P_{i,j,t}} \tag{2}$$

where the last term is a risk premium depending on the perceived probability of default, $P_{i,j,t}$. The minimum interest rate lender i is willing to accept is $r^{min}_{i,j,t} = \frac{\delta P_{i,j,t}}{1 - P_{i,j,t}}$ which is obtained from setting $\mathbb{E}_t R_{i,j,t}$ equal to the return of the outside option. Similarly the borrower will not accept rates higher than $r^{max}_{i,j,t} = r_t$. Note that $\frac{\partial r_{i,j,t}}{\partial P_{i,j,t}} = \delta(1 - \theta_{i,j,t})/(1 - P_{i,j,t})^2 > 0$ and $\frac{\partial^2 r_{i,j,t}}{\partial P^2_{i,j,t}} = 2\delta(1 - \theta)/(1 - P_{i,j,t})^3 > 0$. It is also easy to see that $\frac{\partial r_{i,j,t}}{\partial \theta_{i,j,t}} > 0$ for $r_{i,j,t} > r^{min}_{i,j,t}$ such that lenders with more market power are able to obtain higher rates from their borrowers. Further, $\frac{\partial r_{i,j,t}}{\partial r_t} = \theta_{i,j,t} > 0$ and hence the bilateral interest rate increases with the outside option for borrowing.

[5] In fact, losses may also result from full yet delayed repayment of the principal amount of a loan.

Using the definition of $P_{i,j,t}$ we can rewrite the equilibrium interest rate as a function of the default threshold, the true financial distress variance, and the variance of the perception error as

$$r_{i,j,t} = \theta_{i,j,t} r_t + \delta(1 - \theta_{i,j,t}) \frac{\sigma_{j,t}^2 + \tilde{\sigma}_{i,j,t}^2}{\epsilon_{i,j,t}^2}.$$

Moreover the partial derivatives of this function are given by

$$\frac{\partial r_{i,j,t}}{\partial \sigma_{j,t}} = \frac{\delta(1-\theta_{i,j,t})2\sigma_{j,t}}{\epsilon_{i,j,t}^2} > 0 \quad \text{and} \quad \frac{\partial^2 r_{i,j,t}}{\partial \sigma_{j,t}^2} = \frac{2\delta(1-\theta_{i,j,t})}{\epsilon_{i,j,t}^2} > 0$$

$$\text{and similarly} \quad \frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}} = \frac{(1-\theta_{i,j,t})2\delta\tilde{\sigma}_{i,j,t}}{\epsilon_{i,j,t}^2} > 0 \quad \text{and} \quad \frac{\partial^2 r_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}^2} = \frac{2\delta(1-\theta_{i,j,t})}{\epsilon_{i,j,t}^2} > 0.$$

Thus the equilibrium interest rate increases with the uncertainty about counterparty risk. Furthermore, since the first and second derivatives with respect to the threshold parameter are given by

$$\frac{\partial r_{i,j,t}}{\partial \epsilon_{i,j,t}} = -2\delta(1-\theta_{i,j,t}) \frac{\sigma_{j,t}^2 + \tilde{\sigma}_{i,j,t}^2}{\epsilon_{i,j,t}^3} < 0 \quad \text{and} \quad \frac{\partial^2 r_{i,j,t}}{\partial \epsilon_{i,j,t}^2} = 6\delta(1-\theta_{i,j,t}) \frac{\sigma_{j,t}^2 + \tilde{\sigma}_{i,j,t}^2}{\epsilon_{i,j,t}^4} > 0$$

it follow that, ceteris paribus, the interest rate decreases for a larger threshold when default becomes a less likely event.

The partial derivative of the expected return with respect to the perception error variance is

$$\frac{\partial \mathbb{E}_t R_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}^2} = -\delta \frac{\partial P_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}^2} + \frac{\partial 1 - P_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}^2} r_{i,j,t} + (1 - P_{i,j,t}) \frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}^2} = -\frac{\epsilon_{i,j,t}(1+r_t)\theta_{i,j,t}}{(\epsilon_{i,j,t}^2 + \sigma_{j,t}^2 + \tilde{\sigma}_{i,j,t}^2)^2} < 0.$$

These terms show the channels through which increased uncertainty about counterparty risk affects the expected return. First, it decreases $\mathbb{E}_t R_{i,j,t}$ as $\frac{\partial P_{i,j,t}}{\partial \tilde{\sigma}_{i,j,t}^2} >$ and hence loss given default becomes more likely. Second, it increases the risk premium that is obtained if the borrower survives. However, the net effect is negative and thus the expected return decreases for a larger perception error variance.

The preceding analysis reveals that the bilateral equilibrium interest rate under the asymmetric information problem, here parametrized by the perception error variance, is not Pareto efficient. Indeed, we can compute the interest rate and expected return for the perfect information case where $\tilde{\sigma}_{i,j,t}^2 = 0$ (denoted by $r_{i,j,t}^{PI}$ and $\bar{R}_{i,j,t}^{PI}$) and compare it with the asymmetric information case

$$r_{i,j,t} - r^{PI}_{i,j,t} = \frac{\delta(1-\theta_{i,j,t})\tilde{\sigma}^2_{i,j,t}}{\epsilon^2_{i,j,t}} > 0$$
$$\bar{R}^{PI}_{i,j,t} - \bar{R}_{i,j,t} = \frac{\epsilon^2_{i,j,t}(1+r_t)\theta_{i,j,t}\tilde{\sigma}^2_{i,j,t}}{(\epsilon^2_{i,j,t}+\sigma^2_{j,t})(\epsilon^2_{i,j,t}+\sigma^2_{j,t}+\tilde{\sigma}^2_{i,j,t})} > 0$$

which gives the total reduction in per dollar surplus of the loan due to the asymmetric information problem. Moreover, if $r_{i,j,t} > r_t$ while $r^{PI}_{i,j,t} < r_t$, further inefficiencies arise as no interbank loan occurs though it would be a Pareto improvement. This surplus loss can be reduced by the banks' peer monitoring efforts that reduce counterparty risk uncertainty as we will discuss in the next section.

2.4 Peer Monitoring, Counterparty Search and Transaction Volumes

Banks can engage in costly peer monitoring targeted at mitigating asymmetric information problems about counterparty risk. Therefore, we introduce the $(N \times 1)$ vector $m_{i,t} = (m_{i,1,t}, ..., m_{i,N,t})'$ with each $m_{i,j,t} \in \mathbb{R}^+_0$ describing the monitoring activity of bank i directed towards bank j. In particular, $m_{i,j,t}$ denotes the expenditure that bank i incurred in period t in monitoring bank j. The *added information* that bank i acquires about bank j in period t is assumed to be a linear function of the monitoring expenditure in period t and the occurrence of transaction $l_{i,j,t}$ during the trading session t,

$$\phi_{i,j,t} = \phi(m_{i,j,t}, l_{i,j,t}) = \beta_\phi + \beta_{1,\phi} m_{i,j,t} + \beta_{2,\phi} l_{i,j,t}. \tag{3}$$

The added information affects the perception error variance in future periods, compare equation (1). By allowing $\phi_{i,j,t}$ to be a function of both $l_{i,j,t}$ and $m_{i,j,t}$, we distinguish between (costly) active information acquisition such as costly creditworthiness checks and freely obtained information and trust building via repeated interaction.[6]

The possibility of bilateral Nash bargaining between banks i and j as described in the previous subsection occurs only if these two banks have established a contact. This is an

[6] We do not differentiate between screening and monitoring efforts. Monitoring costs are for instance labor cost for credit analysts, etc. Freely obtained information relates for instance to Babus (2011) who argues that by having a established credit relationship lenders learn about reneged or delayed repayments of their borrowers. We assume that banks' monitoring decisions are made subject to the constraint $\beta_{2,\phi} = 0$.

important consequence of the decentralized structure of interbank markets, see Ashcraft and Duffie (2007) and Afonso and Lagos (2012). Therefore we introduce $\eta_{i,j,t} \in \{0,1\}$ that indicates if a *link* between bank i and j is 'open' and negotiations are possible. We model this variable as a product of two independent random variables

$$\eta_{i,j,t} = B_{i,j,t} U_{i,j,t}, \tag{4}$$

where $B_{i,j,t}$ is a Bernoulli random variable with success probability $\lambda_{i,j,t}$ that can be influenced by the search efforts of bank j directed towards lender i

$$B_{i,j,t} \sim \text{Bernoulli}(\lambda_{i,j,t}) \text{ with } \lambda_{i,j,t} = \lambda(s_{j,i,t}). \tag{5}$$

The variable $s_{j,i,t} \in \mathbb{R}_0^+$ captures the search cost incurred by bank j (with a liquidity deficit) when approaching lender i (assuming loans are borrower initiated). We collect all search efforts of bank j in the $(N \times 1)$ vector $s_{i,t} = (s_{1,i,t}, ..., s_{N,i,t})'$. By making search endogenous we differ from the interbank search model by Afonso and Lagos (2012) that assume an exogenous matching function. This step is crucial in modeling observed interbank relationships as banks will prefer to exchange funds with preferred lending partners that offer most favorable credit conditions.

For the search technology we assume that for increasing search efforts two banks are more likely to establish a contact and engage in interest rate negotiations. Specifically, we model the mean of the binary $B_{i,j,t}$ by a logistic function

$$\lambda_{i,j,t} = \lambda(s_{j,i,t}) = \frac{1}{1 + \exp(-\beta_\lambda (s_{j,i,t} - \alpha_\lambda))},$$

with $\beta_\lambda > 0$ and $\alpha_\lambda > 0$. Note that if $\beta_\lambda \to \infty$ this function converges to a step function that corresponds to a deterministic link formation at fix cost α_λ. Also note that for $s_{j,i,t} = 0$ we still have $\lambda_{i,j,t} > 0$. Compare also the extended use of logistic transformations with linear index functions in the discrete choice literature, in particular the trading relationship model by Weisbuch et al. (2000).

The exogenous shock $U_{i,j,t} \in \{0,1\}$ in equation (4) is needed for a technical reason: it ensures that a bank is only paired with at most one counterpart at each time instance. Such a random matching is a common simplification to achieve tractability in structural network modeling as it avoids solving a complicated multilateral bargaining problem at each time, see Babus (2011) amongst others. Note that due to this simplification there is only one open link for each bank at each instance in time.[7]

Finally, the actual bilateral transaction volume at time t is given by

$$y_{i,j,t} = \begin{cases} \zeta_{i,j,t} & \text{if } \eta_{i,j,t} = 1 \wedge r_{i,j,t} \leq r_t \\ 0 & \text{o/w} \ . \end{cases} \tag{6}$$

such that loans are granted if and only if two banks are in contact, negotiation is possible, and the bargaining is successful in the sense that the bilateral equilibrium interest rate is smaller than the outside option. We assume that for positive loan amounts the transaction volumes are given by the largest feasible amount that the banks can exchange given their current balances

$$\zeta_{i,j,t} = \min\{\zeta_{i,t}, -\zeta_{j,t}\}\mathbb{I}(\zeta_{i,t} \geq 0)\mathbb{I}(\zeta_{j,t} \leq 0). \tag{7}$$

Hence, for sufficiently good risk assessment, the interest rates are directly affected by monitoring effort of bank i and matching only by search efforts of bank j, i.e. the volumes of granted loans are exogenously determined by $\zeta_{i,j,t}$. Thus we abstract from credit rationing on the intensive margin of volumes. The model hence focuses on explaining the extensive margin and the variation in interest rates. Note also that we abstract from liquidity hoarding for precautionary reasons, see Acharya and Merrouche (2010), and solely concentrate on the role of counterparty risk uncertainty.

2.5 Profit Maximization and Optimal Dynamic Monitoring and Search

Each bank faces the dynamic problem of allocating resources to monitoring counterparties and choosing which bank to approach for transaction to maximize expected discounted payoffs of interbank trading. The objective function of each bank $i \in \{1, ..., N\}$ thus

[7]For a given data frequency (in our analysis daily) we therefore draw multiple intra-period liquidity shocks such that multiple links and counterparties over the day are possible. Controls and other variables than liquidity shocks are assumed to change on a daily frequency though. This assumption is merely made for computational convenience.

takes the same form

$$\max_{\{m_{i,t} s_{i,t}\}} \mathbb{E}_t \sum_{s=t}^{\infty} \left[\prod_{v=0}^{s} \left(\frac{1}{1+r_v} \right) \pi_{i,s}(m_{i,t}, s_{i,t}) \right],$$

where $\pi_{i,t}$ denotes the interbank profit of bank i at time t; r_t denotes the riskless lending rate at time t that is used for discounting. Note that the intertemporal problem is made operational by conditioning on the equilibrium interest rates $r_{i,j,t}$ characterized in section 2.3. Hence, in this section these interest rates appear as a restriction on the optimization instead of an argument of the objective function. Assuming a fixed riskless interest rate $r_t = r \; \forall t \in \mathbb{N}$, the objective function above reduces to,

$$\max_{\{m_{i,t}, s_{i,t}\}} \mathbb{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \pi_{i,s}(m_{i,s}, s_{i,s}). \tag{8}$$

The only unspecified function in the objective function above is the profit function $\pi_{i,s}(m_{i,s}, s_{i,s})$. Naturally, period t interbank profits of bank i written in terms of surplus over the outside options are given by[8]

$$\pi_{i,t} = \sum_{j=1}^{N} \bar{R}_{i,j,t} y_{i,j,t} + (r_t - r_{j,i,t}) y_{j,i,t} - m_{i,j,t} - s_{i,j,t},$$

The true dynamic stochastic constrained optimization problem of bank i can thus be formulated as the maximization in (8) subject to the restrictions imposed by the structure laid down in Sections 2.1-2.4. Unfortunately however, it is computationally infeasible to work with the exact solution to this optimization problem for two different reasons.

First, attempting to use N different policy functions for the N different banks would make simulation and estimation prohibitively time-consuming even in large computer clusters. We thus impose that all banks make use of a 'central' policy function derived at the central distribution of the liquidity shocks $\mathcal{N}(\mu_\mu, \mu_\sigma)$. This allows us to derive a unique policy function that can be used by all banks to map state variables into decision variables. It is important to highlight that each bank still makes unique decisions that reflect the magnitude of the liquidity shocks faced by the bank at any moment of time

[8]Note here that default does not enter banks' objective functions ($\bar{R}_{i,j,t} y_{i,j,t}$ not $R_{i,j,t} y_{i,j,t}$) and is only considered in the pricing of interbank loans. While once could incorporate actual bank default into the model this is not essential for explaining the basic mechanisms of peer monitoring and counterparty search.

as well as its unique history in terms of both state and control variables. It is only the map from state to control variables that is common to the 'central' map.

Second, the lack of smoothness of the dynamic optimization problem prevents us from obtaining analytic optimality conditions. Although numerical solutions are in theory possible, these would once again make simulation and estimation prohibitively time-consuming. This occurs because the optimization problem is not smooth due to the presence of step functions in the construction of $y_{i,j,t}$. Hence, it cannot be solved by any of the familiar methods from calculus of variations (Euler-Lagrange), dynamic programming (Bellman equation), or optimal control theory (Hamiltonian). We consider therefore an approximate smooth problem where we replace the step functions by a continuously differentiable function.[9] This allows us to use the calculus of variations that is well understood and the most widely applied method to solve constrained dynamic stochastic optimization problems in structural economics; see e.g. Judd (1998) and DeJong and Dave (2006). We thus obtain Euler equations that constitute approximate solutions to the original non-differentiable model.

Substituting out all definitions except for the law of motion for $\tilde{\sigma}^2_{i,j,t}$, we can write the Lagrange function with multiplier $\mu_{i,j,t}$ given by

$$\mathcal{L} = \mathbb{E}_t \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \sum_{j=1}^{N} \pi_{i,j,t}(m_{i,j,t}, s_{i,j,t}, \tilde{\sigma}^2_{i,j,t}) + \mu_{i,j,t}(\xi(m_{i,j,t}, \tilde{\sigma}^2_{i,j,t}) - \tilde{\sigma}^2_{i,j,t+1}),$$

where we made the arguments explicit that can be influences by bank i's decision. The Euler equations that establish the first-order-conditions to the infinite-horizon nonlinear dynamic stochastic optimization problem can then be obtained by optimizing the Lagrange function w.r.t. to the control variables and the dynamic constraints, see e.g. Heer and Maußner (2005). Under usual regularity conditions the integration and differentia-

[9] Specifically, we use a logistic functions $l(x) = \frac{1}{1+\exp(-\beta_x x)}$ with large scale parameter β_x to approximate the step function $\mathbb{I}\,(x \geq 0)$. Note that for a growing scale parameter the logistic transformation approximates the step function arbitrary well.

tion steps can be interchanged and we obtain

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial m_{i,j,t}} = 0 \;&\Leftrightarrow\; \mathbb{E}_t\Big[\frac{\partial \pi_{i,j,t}}{\partial m_{i,j,t}} + \mu_{i,j,t}\frac{\partial \xi_{i,j,t}}{\partial m_{i,j,t}}\Big] = 0 \\
\frac{\partial \mathcal{L}}{\partial \tilde{\sigma}^2_{i,j,t+1}} = 0 \;&\Leftrightarrow\; \mathbb{E}_t\Big[-\mu_{i,j,t} + \frac{1}{1+r}\left(\frac{\partial \pi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}} + \mu_{i,j,t+1}\frac{\partial \xi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}}\right)\Big] = 0 \\
\frac{\partial \mathcal{L}}{\partial s_{i,j,t}} = 0 \;&\Leftrightarrow\; \mathbb{E}_t\Big[\frac{\partial \pi_{i,j,t}}{\partial s_{i,j,t}} + \frac{1}{1+r}\frac{\partial \pi_{i,j,t+1}}{\partial s_{i,j,t}}\Big] = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_{i,j,t}} = 0 \;&\Leftrightarrow\; \mathbb{E}_t\Big[\tilde{\sigma}^2_{i,j,t+1} - \xi(\phi_{i,j,t}, \tilde{\sigma}^2_{i,j,t})\Big] = 0
\end{aligned}
$$

for all counterparties $j \neq i$ and all t. Substituting out the Lagrange multipliers and taking fixed values at time t out of the expectation gives the Euler equation for the optimal monitoring path,

$$
\frac{\partial \pi_{i,j,t}}{\partial m_{i,j,t}} = \frac{1}{1+r}\frac{\partial \xi_{i,j,t}}{\partial m_{i,j,t}}\mathbb{E}_t\left(\frac{\partial \xi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}}\frac{\frac{\partial \pi_{i,j,t+1}}{\partial m_{i,j,t+1}}}{\frac{\partial \xi_{i,j,t+1}}{\partial m_{i,j,t+1}}} - \frac{\partial \pi_{i,j,t+1}}{\partial \tilde{\sigma}^2_{i,j,t+1}}\right). \tag{9}
$$

that equates marginal cost and discounted expected future marginal benefits of monitoring. Unlike monitoring expenditures search becomes effective in the same period it is exerted and does not directly alter future matching probabilities. Thus the first-order condition for the optimal search path is given by

$$
\frac{\partial \pi_{i,j,t}}{\partial s_{i,j,t}} = -\mathbb{E}_t\frac{\partial \pi_{i,j,t}}{\partial s_{i,j,t}}, \tag{10}
$$

leading to the usual marginal cost equals expected marginal benefits in each period without any discounting. Note that because the first-order conditions hold for all $j \neq i$ and the marginal cost of monitoring and search is the same across all j, the conditions also imply that (discounted) expected marginal profits of monitoring and search must be the same across different banks j.

The transversality condition for the dynamic problem is obtained as the limit to the endpoint condition from the corresponding finite horizon problem and requires that

$$
\lim_{T\to\infty}\mathbb{E}_t\left[\left(\frac{1}{1+r}\right)^{T-2}\frac{\partial \pi_{i,j,T-1}}{\partial m_{i,j,T-1}} - \left(\frac{1}{1+r}\right)^{T-1}\frac{\partial \pi_{i,j,T}}{\partial \tilde{\sigma}^2_{i,j,T}}\frac{\partial \xi_{i,j,T-1}}{\partial m_{i,j,T-1}}\right] = 0.
$$

Thus in the limit the expected marginal cost of investing in monitoring must be equal to the expected marginal return. Under appropriate concavity conditions on the objective function, the Euler equation and the transversality condition are sufficient conditions for an optimal solution. Note that we have ignored the non-negativity constraints $m_{i,j,t} \geq 0$ and $s_{i,j,t} \geq 0$ and hence assumed an interior solution.

2.6 Simulating from Optimal Policy Functions Under Adaptive Expectations

Following the bulk of the literature on stochastic dynamic modeling, see e.g. DeJong and Dave (2006) and Ruge-Murcia (2007), we simulate from the structural model by first deriving the optimal policy rules, linearizing them and then adopting an expectation generating mechanism that delivers the reduced form of the model.

2.6.1 Solution to FOCs

The 'central' Euler equations for the optimal monitoring decisions given in (9) can be written as

$$\mathbb{E}_t f(m_{i,j,t}, m_{i,j,t+1}, \tilde{\sigma}^2_{i,j,t}, \tilde{\sigma}^2_{i,j,t+1}, \lambda_{i,j,t+1}, z_{i,j,t+1}) = 0$$

where we use $z_{i,j,t} = U_{i,j,t}\zeta_{i,j,t}$ to collect the exogenous shocks that cannot be influenced by any bank in the network.

We linearize the function f by a first-order Taylor expansion around a point where the network is stable in some sense. We therefore introduce the concept of the *mean steady steady* denoted by $(\tilde{m}_{i,j}, \tilde{m}_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\lambda}_{i,j}, \tilde{z}_{i,j})$. The mean steady-state does not correspond to the steady-state of the system where all shocks are set to zero, but it is obtained from setting $z_{i,j,t}$ to $\tilde{z}_{i,j} := \mathbb{E}z_{i,j,t}$ and $\eta_{i,j,t}$ to $\tilde{\eta}_{i,j} := \mathbb{E}\eta_{i,j}$, and then finding the time-invariant $(\tilde{m}_{i,j}, \tilde{\lambda}_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j})$ such that the Euler equation holds. We do not expand the function f around the usual steady-state because at this point $\hat{\zeta}_{i,j} = 0$ and hence the network is inactive as $\tilde{y}_{i,j} = 0$ and it corresponds to a critical points where all partial derivatives are zero.

In the following expansion we write $h_x := \frac{\partial h(x,y)}{\partial x}$ and use $\hat{x} := x - \tilde{x}$ to denote deviation from mean steady-state values. Applying the first-order Taylor expansion around the mean steady-state gives

$$\begin{aligned} f \approx & \tilde{f} + f_{m_{i,j,t}} \hat{m}_{i,j,t} + f_{m_{i,j,t+1}} \hat{m}_{i,j,t+1} + f_{\tilde{\sigma}^2_{i,j,t}} \hat{\tilde{\sigma}}^2_{i,j,t} \\ & + f_{\tilde{\sigma}^2_{i,j,t+1}} \hat{\tilde{\sigma}}^2_{i,j,t+1} + f_{\lambda_{i,j,t+1}} \hat{\lambda}_{i,j,t+1} + f_{z_{i,j,t+1}} \hat{z}_{i,j,t+1} \end{aligned}$$

where $\tilde{f} := f(\tilde{m}_{i,j}, \tilde{m}_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\tilde{\sigma}}^2_{i,j}, \tilde{\lambda}_{i,j}, \tilde{z}_{i,j})$ and all derivatives are evaluated at the expansion point, i.e. the mean steady-state. Note that $\tilde{f} = 0$ by construction.

We then obtain the approximate Euler equation for monitoring as

$$\mathbb{E}_t \Big[f_{m_{i,j,t}} \hat{m}_{i,j,t} + f_{m_{i,j,t+1}} \hat{m}_{i,j,t+1} + f_{\sigma^2_{i,j,t}} \hat{\tilde{\sigma}}^2_{i,j,t} + f_{\tilde{\sigma}^2_{i,j,t+1}} \hat{\tilde{\sigma}}^2_{i,j,t+1} + f_{\lambda_{i,j,t+1}} \hat{\lambda}_{i,j,t+1} + f_{z_{i,j,t+1}} \hat{z}_{i,j,t+1} \Big] = 0$$

which we rearrange to get the linear policy function

$$m_{i,j,t} = a_m + b_m \tilde{\sigma}^2_{i,j,t} + c_m \mathbb{E}_t \tilde{\sigma}_{i,j,t+1} + d_m \mathbb{E}_t m_{i,j,t+1} + e_m \mathbb{E}_t \lambda_{i,j,t+1} + f_m \mathbb{E}_t z_{i,j,t+1}, \tag{11}$$

where the intercept and slope coefficients are functions of the structural model parameters. This equation shows that the optimal monitoring expenditures of bank i towards bank j depends on the current state of uncertainty, the expected future uncertainty, the expected monitoring intensity in the next period, the expected volume of the loan, and on the expected probability of being contacted by bank j.

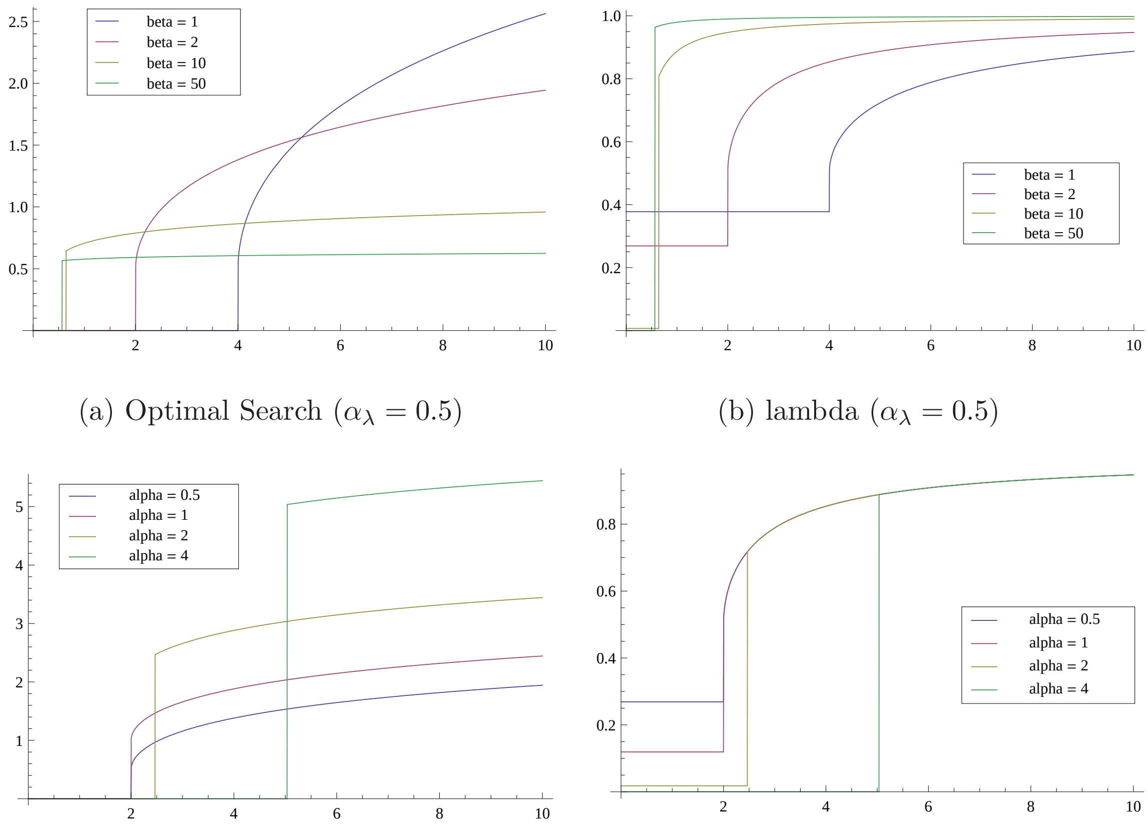
Because $\lambda'(s_{i,j,t})$ is invertible, we obtain an analytical solution to the first order-condition that characterize any interior solution with positive search levels and obtain

$$s(\Delta_{i,j,t}) := 1/\beta_\lambda \log \left(0.5 (\sqrt{\Delta_{i,j,t} \beta_\lambda (\Delta_{i,j,t} \beta_\lambda - 4)} + \Delta_{i,j,t} \beta_\lambda - 2) e^{\alpha_\lambda \beta_\lambda}) \right) \tag{12}$$

for $\Delta_{i,j,t} \beta_\lambda (\Delta_{i,j,t} \beta_\lambda - 4) \geq 0$ where $\Delta_{i,j,t} := \mathbb{E}_t [U_{j,i,t} I_{j,i,t} \zeta_{j,i,t} (r_t - r_{j,i,t})]$ is the expected surplus (relative to the outside option r_t) of bank i when borrowing funds from lender j. The optimal search level of bank i towards bank j at time t is thus given by

$$s_{i,j,t} = \begin{cases} s(\Delta_{i,j,t}) & \text{for} \;\; \Delta_{i,j,t} \lambda(s(\Delta_{i,j,t})) - s(\Delta_{i,j,t}) \geq 0 \\ 0 & \text{for} \;\; \Delta_{i,j,t} \lambda(s(\Delta_{i,j,t})) - s(\Delta_{i,j,t}) < 0. \end{cases} \tag{13}$$

That is for positive expected return net of search cost the solutions satisfies Equation (12). Note that $s_{i,j,t}$ is a non-linear function of the expected surplus $\Delta_{i,j,t}$. Figure 1 depicts the optimal search strategy $s_{i,j,t}$ and implied $\lambda_{i,j,t}$ as a function of $\Delta_{i,j,t}$ for different parameter values of β_λ and α_λ.



(a) Optimal Search ($\alpha_\lambda = 0.5$)

(b) lambda ($\alpha_\lambda = 0.5$)

(c) Optimal Search ($\beta_\lambda = 2$)

(d) lambda ($\beta_\lambda = 2$)

Figure 1: Optimal search effort $s_{i,j,t}$ and implied probability of success $\lambda_{i,j,t}$ as a function of expected surplus $\Delta_{i,j,t}$, for different parameter values of α_λ and β_λ.

As can be seen from the Figure 1 the location and slope parameter affect how fast search efforts decrease, as well as on the actual location when a drop to zero search effort happens and the problem has a corner solution. It also is important to note that $\lambda(0) > 0$ and hence even without search efforts two banks will eventually get connected to each other.

2.6.2 Adaptive Expectations

It is important to highlight that lender i's monitoring level with respect to borrower j depends on the expectation of being approached for transaction by the latter. Similarly borrower j's search level with respect to lender i depends on the expected surplus he can obtain from borrowing from i. This interrelationship between monitoring and search creates the potential of the emergence of bilateral trading relationships where counterparty risk uncertainty is mitigated due to repeated monitoring. Expectations about credit conditions at other banks and about the choice variables of other banks play a crucial role in this mechanism.

We assume that banks form bank-specific *adaptive expectations* about future credit conditions in the market. However, banks are not only uncertain about future variables but also about present and past credit conditions at other banks. Therefore, the decisions of banks are the result of a complex web of expectations about past, present and future trading conditions that change every period as banks search, monitor and ultimately have contact with each other. This is an essential feature of the opaque interbank market structure, where information is not revealed in a central manner, but banks learn about credit condition at other banks only through contact and engaging in trade.

The adoption of adaptive expectations is justified in the first place by the fact that adaptive expectations are much easier to handle, see Mlambo (2012). Indeed, the impossibility of using the deterministic steady-state of the model as an approximation point for perturbation methods render rational expectation solution method impractical. On the contrary, since adaptive expectations are solely dependent on past observations, the numerical nature of the equilibrium point does not present extra difficulties.

The adaptive expectations hypothesis, which dates back to the 1950s, was adopted for modeling purposes as early as Harberger (1960). Contrary to popular belief, in many settings, there exists a collection of very strong econometric evidence supporting the adaptive expectations hypothesis against the rational expectations hypothesis; see e.g. Chow (1989) and Chow (2011). More recently, Evans and Honkapohja (1993) and Evans and Honkapohja (2001) showed that adaptive expectations are not only reasonable, but in many ways the most rational forecast method when the true process is unknown. In this paper we adopt the classical geometrically declining weights for which Chow (1989) and Chow (2011) also finds strong support.

In particular, the adaptive expectation formation of bank i concerning variable $x_{i,j,t}$ draws from past experience according to an exponentially weighted moving average (EWMA)

$$\mathbb{E}_t x_{i,j,t+1} := x^*_{i,j,t} = (1-\lambda_x)x^*_{i,j,t-1} + \lambda_x x_{i,j,t}, \tag{14}$$

where all variables are in deviation from the mean steady-state values. In particular we use this forecasting rule for variables that are always observed by bank i (for instance, $\hat{m}_{i,j,t}$, the own monitoring effort). Due to the decentralized structure a bank however learns about credit condition at *other* banks only once upon contact. We incorporate this crucial feature of decentralized interbank markets by assuming that bank i uses the following forecasting rule,

$$x^*_{i,j,t} = (1-\lambda_x)x^*_{i,j,t-1} + \lambda_x \eta_{i,j,t} x_{i,j,t}. \tag{15}$$

Recall that $\eta_{i,j,t}(s_{j,i,t-1}) = 1$ denotes an open connection. Hence new information about a counterparty is added to the last forecast only if the banks where in contact, otherwise the last forecast is not maintained but discounted by a factor $(1-\lambda_x)$. Thus if bank i and j are not in contact for a long time their expectations converge to the mean steady-state values. In both cases the initial value is assumed to be given by $x^*_0 = x_0$ and the parameter $\lambda_x \in (0,1)$ determines the weight of the new observations at t. The EMWA assumes an AR(1) process for the conditional expectation of $x_{i,j,t+1}$ with innovations $\eta_{i,j,t}x_{i,j,t}$. Note that for a stationary innovations sequence the EMWA forecasts are stationary. Some simple algebra also shows that $x^*_{i,j,t} = \lambda_x \sum_{s=0}^{\infty}(1-\lambda_x)^s \eta_{i,j,t-s}x_{i,j,t-s}$, such that the expectation is a weighted average of all past observations with geometrically decaying weights.

2.6.3 Reduced Form, Stationarity and Ergodicity

Substituting the adaptive expectation mechanism in (14) and (15) into the Euler equation for monitoring in (11) and the optimal search strategy in (13) allows us to re-write the full system in reduced form. The reduced form system takes the form of a nonlinear Markov autoregressive process,

$$\mathbf{X}_t = \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{X}_{t-1}, \mathbf{e}_t)$$

where $\mathbf{G}_{\boldsymbol{\theta}}$ is a parametric vector function that depends on the structural model parameter $\boldsymbol{\theta}$, and $\mathbf{X}_t$ is the vector of all state-variables and control variables (observed or unobserved), and $\mathbf{e}_t$ is the vector of shocks driving the system. These shocks are the bank-specific liquidity shocks, the pair-specific shocks to the perception error variance, and the shocks that determine if a link between any two banks is open and trade is possible. Obtaining the reduced form representation is crucial as it allows us to simulate network paths for both state and control variables under a given structural parameter vector. Furthermore, this model formulation allows us to describe conditions for the strict stationarity and ergodicity of the model that are essential for the estimation theory that is outlined in section 3.

In particular, following Bougerol (1993), we note that under appropriate regularity conditions, the process $\{\mathbf{X}_t\}$ is strictly stationary and ergodic (SE).

LEMMA 1. *For every $\boldsymbol{\theta} \in \Theta$, let $\{\mathbf{e}_t\}_{t\in\mathbb{Z}}$ be an SE sequence and assume there exists a (nonrandom) $\mathbf{x}$ such that $\mathbb{E}\log^+ \|\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t) - \mathbf{x}\| < \infty$ and suppose that the following contraction condition holds*

$$\mathbb{E}\ln \sup_{\mathbf{x}'\neq\mathbf{x}''} \frac{\|\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}', \mathbf{e}_t) - \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}'', \mathbf{e}_t)\|}{\|\mathbf{x}' - \mathbf{x}''\|} < 0 \tag{16}$$

Then the process $\{\mathbf{X}_t(\mathbf{x}_1)\}_{t\in\mathbb{N}}$, initialized at $\mathbf{x}_1$ and defined as

$$\mathbf{X}_1 = \mathbf{x}_1 \quad , \quad \mathbf{X}_t = \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{X}_{t-1}, \mathbf{e}_t) \ \forall\ t \in \mathbb{N}$$

converges e.a.s. to a unique SE solution $\{\mathbf{X}_t\}_{t\in\mathbb{Z}}$ for every $\mathbf{x}_1$, i.e. $\|\mathbf{X}_t(\mathbf{x}_1) - \mathbf{X}_t\| \stackrel{e.a.s.}{\to} 0$ as $t \to \infty$.[10]

The condition that $\mathbb{E}\log^+ \|\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t) - \mathbf{x}\| < \infty$ can be easily verified for any given distribution for the innovations $\mathbf{e}_t$ and any given shape function $\mathbf{G}_{\boldsymbol{\theta}}$. The contraction condition in (16) is however much harder to verify analytically.

Fortunately, the contraction condition can be re-written as

$$\mathbb{E}\log\sup_{\mathbf{x}} \|\nabla\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t)\| < 0 \tag{17}$$

where $\nabla\mathbf{G}_{\boldsymbol{\theta}}$ denotes the Jacobian of $\mathbf{G}_{\boldsymbol{\theta}}$ and $\|\cdot\|$ is a norm. By verifying numerically that this inequality holds at every step $\boldsymbol{\theta} \in \Theta$ of the estimation algorithm, one can ensure that the simulation-based estimation procedure has appropriate stochastic properties.

The contraction condition of Bougerol (1993) in (17) states essentially that the maximal Lyapunov exponent must be negative uniformly in $\mathbf{x}$.

DEFINITION 1. *The maximal Lyapunov exponent is given by $\lim_{t\to\infty} \frac{1}{t}\log\max_i \Lambda_{i,t} =$*

[10] A stochastic sequence $\{\xi_t\}$ is said to satisfy $\|\xi_t\| \stackrel{e.a.s.}{\to} 0$ if $\exists\ \gamma > 1$ such that $\gamma^t\|x_t\| \stackrel{a.s.}{\to} 0$.

$\mathbb{E} \log \max_i \Lambda_{i,t}$ *where* $\Lambda_{i,t}$*'s are eigenvalues of the Jacobian matrix* $\nabla \mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}_t, \mathbf{e}_t)$.

A negative Lyapunov exponent ensures the stability of the network paths. Table 1 uses the Jacobian of the structural dynamic system $\mathbf{G}_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{e}_t)$ to report numerical calculations of the maximal Lyapunov exponent of our dynamic stochastic network model at the parameters $\boldsymbol{\theta}_0$ and $\hat{\boldsymbol{\theta}}_T$ described in Table 3 of Section 3.3 below. These points in the parameter space corresponds to the starting point for the estimation procedure described in Section 3 and the final estimated point.

Table 1: Lyapunov stability of the dynamic stochastic network model

Parameter vector	$\boldsymbol{\theta}_0$	$\hat{\boldsymbol{\theta}}_T$
Lyapunov exponent	-0.0411	-0.0147

Despite the higher degree of persistency at $\hat{\boldsymbol{\theta}}_T$ compared to $\boldsymbol{\theta}_0$ (higher Lyapunov exponent), the contraction condition is satisfied in both cases as the maximal Lyapunov exponent is negative. This ensures that both $\boldsymbol{\theta}_0$ and $\hat{\boldsymbol{\theta}}_T$ generate stable network paths.

3 Parameter Estimation

We turn now to the estimation of the structural parameters using data from the Dutch overnight interbank lending market. Due to the complexity of the model (non-linearity and non-standard distributions) the likelihood function is not known even up to a constant, rendering ML or Bayesian methods impossible. We therefore resort to the simulation-based method of indirect inference that is building on an appropriate set of auxiliary statistics to estimate the structural model parameters.

3.1 Auxiliary Statistics and Indirect Inference

Following the principle of indirect inference introduced in Gourieroux et al. (1993), we estimate the vector of parameters $\boldsymbol{\theta}_T$ by minimizing the distance between the auxiliary statistics $\hat{\boldsymbol{\beta}}_T$ obtained from the observed data $X_1, ..., X_T$, and the average of the auxiliary statistics $\tilde{\boldsymbol{\beta}}_{TS}(\theta) := (1/S) \sum_{s=1}^{S} \tilde{\boldsymbol{\beta}}_{T,s}(\theta)$ obtained from S simulated data sets $\{\tilde{X}_{1,s}(\boldsymbol{\theta}), ..., \tilde{X}_{T,s}(\boldsymbol{\theta})\}_{s=1}^{S}$ generated under $\theta \in \Theta$. Formally the indirect inference estimator is thus given as

$$\hat{\boldsymbol{\theta}}_T := \underset{\boldsymbol{\theta} \in \Theta}{\operatorname{argmax}} \left\| \hat{\boldsymbol{\beta}}_T - \frac{1}{S} \sum_{s=1}^{S} \tilde{\boldsymbol{\beta}}_{T,s}(\boldsymbol{\theta}) \right\|,$$

where Θ denotes the parameter space of $\boldsymbol{\theta}$. Under appropriate regularity conditions this estimator is consistent and asymptotically normal. In particular, consistency holds as long as, for given $S \in \mathbb{N}$, the auxiliary statistics converge in probability to singleton limits $\hat{\boldsymbol{\beta}}_T \xrightarrow{p} \boldsymbol{\beta}(\boldsymbol{\theta}_0)$ and $\tilde{\boldsymbol{\beta}}_{T,s} \xrightarrow{p} \boldsymbol{\beta}(\boldsymbol{\theta})$ as $T \to \infty$ and the so-called *binding function* $\boldsymbol{\beta} : \Theta \to \mathcal{B}$ is injective. Convergence in probability is precisely ensured through the application of a law of large numbers for strictly stationary and ergodic data; see e.g. White (2001). Similarly, asymptotic normality is obtained if the auxiliary statistics $\hat{\boldsymbol{\beta}}_T$ and $\tilde{\boldsymbol{\beta}}_{T,s}$ are asymptotically normal; see Gourieroux et al. (1993) for details. By application of a central limit theorem, see e.g. White (2001), the asymptotic normality of the auxiliary statistics can again be obtained by appealing to the strict stationarity and ergodicity of both observed and simulated data.

The injective nature of the binding function is the fundamental identification condition which ensures that the structural parameters are appropriately described by the auxiliary statistics. This condition cannot be verified algebraically since the binding function is analytically intractable. However, identification will be ensured as long as the set of auxiliary statistics describes conveniently both observed and simulated data.

The auxiliary statistics are hence selected so as to characterize as well as possible the interbank market represented by the network of bilateral loans and associated volumes and interest rates. Specifically, we explore the auto-covariance structure, higher-order moments, and a number of network statistics as auxiliary statistics. The use of means, variances and auto-covariances is in line with the bulk of the literature concerned with the estimation of structural models such as Dynamic Stochastic General Equilibrium models; see e.g. DeJong and Dave (2006) and Ruge-Murcia (2007). The use of higher-order moments such as measures of skewness and kurtosis is justified by the nonlinearity of the model and the fact that some structural parameters might be well identified by such statistics.

Besides these standard auxiliary statistics, we base the indirect inference estimator of the network model on auxiliary statistics that characterize specifically the network structure of the interbank market.[11]

First, we consider global network statistic. In particular, the *density* defined as the ratio of the actual to the potential number of links is a standard measures of the con-

[11]The characterization of interbank markets by means of descriptive network statistics has become popular during recent years as studies, for instance Bech and Atalay (2009), Iori et al. (2008), van Lelyveld and in 't Veld (2012) and Abbassi et al. (2013) for the Euro area.

nectivity of a network. A low density characterizes a sparse network with few links. The *reciprocity* measures the fraction of reciprocal links in a directed network. For the interbank market this relates to the degree of mutual lending between banks. The *stability* of a sequence of networks refers to the share of links that do not change between two adjacent periods. Note that all three statistics are bounded between zero and one.

Second, we include bank (node) level network statistics. The (unweighted) *in-degree* of a bank is defined as the number of lenders it is borrowing funds from, the (unweighted) *out-degree* as the number of borrowers is is lending funds to. We summarize this bank level information using the mean and standard deviation of the (in-/out-) degree distribution. The (local) *clustering coefficient* of a node quantifies how close its neighbors are to being a clique (complete graph). In the interbank network it measure how many of a bank's counterparties have mutual credit exposures to each other. We compute the clustering coefficients for directed networks as proposed by Fagiolo (2007) and consider the mean and standard deviation as summary statistics of the cross-sectional distribution.

Third, we compute simple bilateral network statistics that measure the intensity of a bilateral trading relationship based on a rolling window. Similar to Furfine (1999) and Cocco et al. (2009), we compute the number of loans given from bank i to bank j during the last week and denote this variable by $l^{rw}_{i,j,t}$. Further, for any bank pair we compute the lender and borrower preference index ($LPI^{rw}_{i,j,t}$ and $BPI^{rw}_{i,j,t}$) during the rolling window that measure the relative importance of banks forming a pair in terms of lending and borrowing portfolio concentration. We then compute a cross-sectional correlation between these relationship variables and loan outcomes at time t (decision to grant a loan and interest rate).

All described network statistics are computed for the network of interbank lending at each time period t such that we obtain a sequence of network statistics. We then obtain the unconditional means, variance and auto-correlation of these sequences as auxiliary statistics and base the parameter estimations on the values of the auxiliary statistics only. In appendix B we provide the formulae of the described network statistics. For further details on network statistics in economics see Jackson (2008).

3.2 The Data

The original raw data used in estimation comprises the bilateral lending volumes and interest-rates practiced on a daily basis in the over-night lending market between Dutch banks. In particular, we make use of a confidential dataset of interbank loans that has been compiled by central bank authorities based on payment records in the European large value payment system *Target 2*. The panel of interbank loans has been inferred using a modified and improved version of the algorithm proposed by Furfine (1999), for details on the data set and methodology see Heijmans et al. (2011) and Arciero et al. (2013). The data used in the estimation spans the period between 1 February 2008 until 20 March 2011 containing $T = 810$ observations of *Target 2* trading days. This period includes naturally several periods of severe financial stress, in particular the fall of Lehman in September 2008 has been a major shock to counterparty risk uncertainty and the beginning of the European sovereign crisis in 2010.

Here we focus on the $N = 50$ largest banks according to their average trade volume and trade frequency throughout the sample. As a result, the observed data from which auxiliary statistics are obtained consists in essence of three $N \times N \times T$ arrays with elements $l_{i,j,t}$, $y_{i,j,t}$ and $r_{i,j,t}$ observed at daily frequency. The arrays for $y_{i,j,t}$ and $r_{i,j,t}$ contain missing values if and only if $l_{i,j,t} = 0$. The following table provides an overview of summary statistics of the daily data. As we shall see in Table 6 some of the reported statistics are used as auxiliary statistics in the indirect inference procedure.[12] See appendix C for further summary statistics of the data.

Table 2: Descriptive statistics. The table reports moment statistics for different sequences of network statistics and cross-sectional correlations that characterize the sequence of observed Dutch unsecured interbank lending networks. The statistics are computed on a sample of daily frequency from 1 February 2008 to 30 April 2011.

Statistic	Mean	Std	Autocorr
Density	0.0212	0.0068	0.8174
Reciprocity	0.0819	0.0495	0.2573
Stability	0.9818	0.0065	0.8309
Mean out-/indegree	1.0380	0.3323	0.8174
Mean clustering	0.0308	0.0225	0.4149
$\text{Corr}(r_{i,j,t}, l^{rw}_{i,j,t-1})$	-0.0716	0.1573	0.4066
$\text{Corr}(l_{i,j,t}, l^{rw}_{i,j,t-1})$	0.6439	0.0755	0.4287
Mean log-volume	4.1173	0.2818	0.4926
Mean rate	0.2860	0.3741	0.9655

[12]The vector of auxiliary statistics can be written as $\hat{\boldsymbol{\beta}}_T = 1/T \sum_{t=1}^{T} g(l_t, y_t, r_t)$ where $g(\cdot)$ is a continuous vector function of the data at time t. To take into account for well documented end-of-maintenance period effects, we deseasonalize the sequence $\{g(l_t, y_t, r_t)\}_t$ by regressing them on the last three days of the reserve period.

Note that: (i) the moments of traded volumes are for values stated in (logarithm of) millions of euros; (ii) the moments of interest rates are reported in percentage points per annum above the ECB deposit facility rate (lower bound of the interest rate corridor); (iii) the daily interbank network is very sparse with mean density 0.02 (on average 1.04 lenders and borrowers) and low clustering; (iv) the distribution of interest rates, volumes, degree centrality and clustering are highly skewed, see the appendix.

It is also important to highlight here that the high auto-correlation of the density, the high stability of the network as well as the positive expected correlation between the probability of bilateral present link and past lending activity can be seen as evidence of the construction of 'trust relations' between banks and shows that past trades affect future trading opportunities. Similarly, the negative expected correlation between past lending activity and present interest rates provide evidence of reduction in perceived risk that may be justified by monitoring efforts as postulated by the proposed structural model.

3.3 Estimation Results

We next turn to the estimation results of the interbank network model. Table 3 below shows the point estimates, standard errors and 90% confidence intervals of the structural parameters using a quadratic objective function and $S = 12$ simulated network paths each of length $T^* = 5000$.[13] The inference is based on the auxiliary statistics $\hat{\boldsymbol{\beta}}_T$ that are reported in Table 6 and which are computed on a sample of daily frequency from 1 February 2008 to 30 April 2011 ($T = 810$). Appendix D contains details on the choice of the objective function. Note that some of the structural parameters are calibrated as these are not identified by the data. For example, it is clear that several combinations of β_σ, $\beta_{1,\phi}$ and $\beta_{2,\phi}$ imply the same distribution for the data, and hence, also for the auxiliary statistics. The same applies for ϵ and σ.[14] Naturally, standard errors and confidence intervals are not provided for calibrated parameters. An alternative, calibrated

[13]In theory, selecting a larger S would allow us to reduce the estimation uncertainty and hence obtain smaller confidence bounds. However, in practice, a larger S proved infeasible due to the spectacular computational requirements of this large network model. For example, the innovations $\{\mathbf{e}_t\}$ required to simulate a single path of $\{\mathbf{X}_t\}$ amount already to a total of 340 MB memory. Clearly, such heavy machinery imposes practical bounds on the ability to estimate under large S.

[14]We also fix the width of the ECB interest rate corridor to the observed average during the period and set the discount rate to the ECB target rate. Because we do not observe actual bank default we fix the common default threshold $\epsilon_{i,j,t} = \epsilon$ and the common true variance of the financial distress $\sigma^2_{j,t} = \sigma^2$. Further we assume a common bargaining power $\theta_{i,j,t} = \theta$ and set $\delta = 1$.

parameter vector ($\boldsymbol{\theta_a}$) is presented for comparison to the indirect inference estimates.

Table 3: Estimated parameter values. The table reports the estimated structural parameters ($\hat{\boldsymbol{\theta}}_T$) and corresponding standard errors and 90% confidence bounds. For calibrated parameters no standard errors and confidence bounds are reported. The II estimator is based on $S = 12$ simulated network sequences of length $T^* = 5000$. The parameter $\boldsymbol{\theta}_a$ represents an alternative, calibrated model parametrization. Note also that $\sigma^*_\mu = \log(\sigma_\mu)$.

Structural parameters		Calibrated $\boldsymbol{\theta}_a$	Estimated $\hat{\boldsymbol{\theta}}_T$	St.Errors ste($\hat{\boldsymbol{\theta}}_T$)	90% Bounds LB	UB
Added information	α_ϕ	-1.5000	-1.5000	-	-	-
	$\beta_{\phi,1}$	0.0000	8.2949	0.0731	8.1747	8.4152
	$\beta_{\phi,2}$	0.0000	0.2579	0.0107	0.2404	0.2754
Perception error	α_σ	0.0020	0.0020	0.7063	-1.1597	1.1637
	β_σ	2.0000	2.0000	-	-	-
	γ_σ	0.6233	4.6233	0.0185	4.5928	4.6538
	δ_σ	0.0164	1.1636	0.0352	1.1056	1.2215
Search technology	α_λ	0.0041	0.4089	0.0288	0.3615	0.4563
	β_λ	2.1957	26.1957	0.0043	26.1886	26.2028
Liquidity shocks	μ_μ	0.0000	0.0000	-	-	-
	σ^*_μ	2.4403	2.4403	0.0126	2.4196	2.4610
	μ_σ	3.4122	3.4122	0.5232	2.5517	4.2727
	σ_σ	2.5683	2.5683	0.2051	2.2310	2.9056
Expectations	λ^m	0.7728	0.7728	0.0183	0.7426	0.8029
	λ^l	0.9854	0.9854	0.0806	0.8528	1.1179
	λ^z	0.8027	0.8027	0.2698	0.3589	1.2465
	λ^r	0.8916	0.8916	0.3058	0.3887	1.3946
	λ^σ	0.6761	0.6761	0.0948	0.5201	0.8320
Bargaining lender	θ	0.5890	0.5890	0.2114	0.2412	0.9367
CB corridor width	r	1.5000	1.5000	-	-	-
Default threshold	ϵ	3.0000	3.0000	-	-	-
Financial distress	σ	0.1000	0.1000	-	-	-
Discount rate	r	0.0000	0.0000	-	-	-
Scale logistic	β_I	200.00	200.00	-	-	-

The parameter estimates reported in Table 3 are interesting in several respects. First, both estimated slope coefficients of ϕ are positive indicating the relevance of both peer monitoring and past experience on the added information that reduces counterparty risk uncertainty. Because β_σ is larger than $1 + \exp(-3)$ clearly without any monitoring or trading the autoregressive parameter in the evolution of the perception error variance is larger than 1 and uncertainty increases. Note the large scaling of the shocks to $\tilde{\sigma}$ such that uncertainty is continuously pushed upwards, leading to prohibitively large perceived probabilities of default that prevent interbank lending in the absence of any trading or monitoring.

Second, the positive estimate for α_λ and β_λ shows that counterparty search is a crucial feature in the formation of interbank networks. It also highlights the effect of expected profitability and counterparty selection. In particular, the positive estimate for β_λ suggests that links are not formed at random but are strongly influenced by banks' search towards preferred counterparties. In this respect, the positive point estimate for the expectation parameter of available interest rates, λ^r, and available loan volumes, λ^z, indicate persistent expectations about credit conditions. Similarly the values of λ^l and λ^m indicate a relatively strong persistence in the expectations of monitoring and the expectation of being contacted by a specific borrower. These persistent expectation eventually contribute to the high persistence of the simulated interbank network.

Third, the estimated values of the central liquidity distribution that parameterizes bank heterogeneity in liquidity shocks point towards considerably different variances in liquidity shocks. The estimated log normal distribution implies that there are few banks with very large liquidity shock variances that are very active players in the market. Moreover the notion that some banks are structural liquidity providers or supplier is supported by the positive estimate of the variance parameter of the mean. These findings are in line with previous empirical results, see Furfine (1999) and the core-periphery structure found by Craig and von Peter (2010).

Table 4: Coefficients of linear policy rule for optimal monitoring

Variable	$\tilde{\sigma}_{i,j,t}$	$\mathbb{E}_t\tilde{\sigma}_{i,j,t+1}$	$\mathbb{E}_t m_{i,j,t+1}$	$\mathbb{E}_t\lambda_{i,j,t+1}$	$\mathbb{E}_t\zeta_{i,j,t+1}$
Coefficient	0.0431	-0.0223	-0.0165	0.0541	0.0479

In Table 4 we report the coefficients of the linear policy rule for the optimal monitoring levels as implied by the estimated parameters, expressed in deviations from their mean steady-state values. It is particular noteworthy that the optimal monitoring level towards a particular bank depends positively on the expected probability of being approached by this bank to borrow funds during future trading sessions. Indeed this positive coefficient and the significantly positive effect of search on link formation creates the interrelation between monitoring and search as the source of interbank relationship lending. Moreover note that the current state of uncertainty positively affects monitoring during this period, higher expected future uncertainty however reduces these efforts. Also notice the positive coefficient of the amount of granted loans. Hence, banks prefer to monitor those counterparties where they expect larger volumes as the surplus that can be generated by reducing credit risk uncertainty is larger. This implies that banks with on average opposite liquidity shocks will monitor each other more closely as they will on average exchange funds of larger quantity. The negative coefficient of expected future monitoring leads to the usual inter-temporal smoothing of expenditures that is well known from dynamic economic modeling.

Table 5: Testing economic hypotheses

Economic hypothesis	H_0	$\hat{\theta}$	t-stat	Decision[1]
Monitoring has no effect on information	$\beta_{\phi,1} = 0$	8.2949	113.47	reject
Search has no effect on link probability	$\beta_{\lambda} = 0$	26.1957	6092.02	reject
No liquidity shock heterogeneity in mean	$\sigma^*_{\mu} = 0$	2.4403	193.67	reject
No liquidity shock heterogeneity in variance	$\sigma_{\sigma} = 0$	2.5683	12.52	reject

[1] Decisions are based on 5% significance level

Using the estimated structural model where parameters are associated to clear economic concepts, we can test policy-relevant economic theories about the interbank market. Table 5 provides a summary of hypotheses about monitoring, search and bank heterogeneity, along with the respective test statistics and test results. Based on a 5% significance level we can reject (i) the hypothesis that monitoring has no effects on the added information, and (ii) the hypothesis that link formation does not depend on banks' search effort. Thus the data provides evidence of the importance of search and monitoring as two key driving forces in the formation of interbank networks. In particular, the results empirically support the ECB's view that peer monitoring of counterparty risk is part of unsecured interbank lending and thereby may justify a decentralized market solution. Similarly, the result that search has a significant impact on link formation highlights the non-random nature of interbank relationships as banks choose those trading partners that offer them best credit conditions. Furthermore, we can reject the hypothesis that banks have homogeneous liquidity shocks, in both the mean and the variance of the liquidity distribution. Thus the structure of liquidity shocks in the banking system is an important reason for the observed interbank network structure and bilateral lending relations.

We next analyze the model fit and auxiliary statistics. Table 6 shows how the estimated structural parameter vector $\hat{\boldsymbol{\theta}}_T$ produces an accurate description of the data when compared to the alternative calibrated parameter vector $\boldsymbol{\theta}_a$ stated in Table 3 where monitoring has no role on information and search has a relatively less important role.

Table 6: Auxiliary statistics. The table reports the values of the observed auxiliary statistics $\hat{\boldsymbol{\beta}}_T$ used in the indirect inference estimation along with the HAC robust standard errors. The simulated average of the auxiliary statistics $\tilde{\boldsymbol{\beta}}_{TS}$ is shown for the estimated parameter vector and the alternative calibration. The observed statistics are computed on a sample of daily frequency from 1 February 2008 to 30 April 2011 of size $T = 810$.

	Observed		Simulated	
Auxiliary statistic	$\hat{\boldsymbol{\beta}}_T$	ste($\hat{\boldsymbol{\beta}}_T$)	$\tilde{\boldsymbol{\beta}}_{TS}(\boldsymbol{\theta}_a)$	$\tilde{\boldsymbol{\beta}}_{TS}(\hat{\boldsymbol{\theta}}_T)$
Density (mean)	0.0212	0.0026	0.3517	0.0234
Reciprocity (mean)	0.0819	0.0028	0.3248	0.3807
Stability (mean)	0.9818	0.0025	0.6343	0.9777
Std outdegree (mean)	1.8406	0.0917	7.9152	1.7613
Std indegree (mean)	1.6001	0.0994	7.4772	1.7500
Avg clustering (mean)	0.0308	0.0027	0.3978	0.0887
Std clustering (mean)	0.0880	0.0079	0.0321	0.2061
Corr($r_{i,j,t}, l^{rw}_{i,j,t-1}$) (mean)	-0.0716	0.0112	-0.0001	-0.0726
Corr($l_{i,j,t}, l^{rw}_{i,j,t-1}$) (mean)	0.6439	0.0106	0.3542	0.5404
Corr($l_{i,j,t}, LPI^{rw}_{i,j,t-1}$) (mean)	0.4085	0.0085	0.2093	0.1520
Corr($l_{i,j,t}, BPI^{rw}_{i,j,t-1}$) (mean)	0.4379	0.0090	0.2318	0.1480
Avg log volume (mean)	4.1173	0.0516	2.8292	4.3683
Std log volume (mean)	1.6896	0.0200	1.2778	2.0391
Avg interest rates (mean)	0.2860	0.1330	1.1276	1.1822
Std interest rates (mean)	0.1066	0.0141	0.0013	0.0635
Skew interest rates (mean)	0.6978	0.5295	1.8315	1.8091
Density (std)	0.0068	0.0004	0.0099	0.0025
Corr(density,stability)	-0.7981	0.0275	-0.1644	-0.6239
Objective function value			33.5964	1.2390
Euclidean norm $\lVert\hat{\boldsymbol{\beta}}_T - \tilde{\boldsymbol{\beta}}_{TS}\rVert$			8.7336	1.5973
Sup norm $\lVert\hat{\boldsymbol{\beta}}_T - \tilde{\boldsymbol{\beta}}_{TS}\rVert_\infty$			6.0747	1.1113

First note the remarkable improvement in model fit compared to the calibrated example, brought about by the indirect inference estimation, as judged by (i) the value of the (log) criterion function that is about 27 times smaller for the estimated model, and (ii) the comparison between auxiliary statistics obtained from observed, data simulated at the calibrated parameter, and data simulated at the estimated parameters. For instance, both the Euclidean norm and the sup norm of the difference between observed and simulated auxiliary statistics are more than 5 times larger under the calibration without monitoring.

A closer look at individual auxiliary statistics reveals several interesting features of the estimated model. First, it is important to highlight the significant improvement in the fit of the density compared to the calibrated example. In fact the estimated model matches very well the sparsity of the Dutch interbank network with a density of 0.02 very close to the observed one. Likewise, the present structural model provides a very accurate description of the high stability of the network and the standard deviations of the degree distributions. Similarly the average clustering coefficient is improved considerably compared to the calibrated model and matches the data rather well. Hence the proposed structural economic model is able to generate a lending structure very similar to the observed one, that is sparse and highly persistence. Both characteristics can be attributed to the structure of liquidity shocks across banks and asymmetric information problems about counterparty risk and search frictions. Indeed, the calibrated parameter $\boldsymbol{\theta}_a$ parametrized a setting where counterparty risk uncertainty is lower and is more equally distributed in the cross-section, and loans with different counterparties are possible without monitoring efforts or past trading experience.

Second, and key to our analysis, the estimated structural model is able to generate patterns of relationship lending where banks repeatedly interact with each other and trade at lower interest rates. In particular, the model very accurately matches the observed correlation of interest rates and past trading of -0.072. As reported in Table 4 monitoring efforts positively depend on the expectation about being approached by a specific borrower. Once a contact between two banks is established, banks positively adjust their expectation and increase monitoring. This has dampening effects on the bilateral interest rate level and thereby further attracts the borrower leading to yet increased expectations about a contact. The role of counterparty selection and monitoring as crucial agents behind the observed dynamic structure of the interbank market is also confirmed by comparing the fit of the auxiliary statistics simulated under the calibrated parameter with those of the estimated parameter. We see that the correlation between past trading and current interest rate increases by a factor of 700 under the estimated parameter. Also note the positive correlation of 0.54 between past and current bilateral lending activity, that measures the stability of bilateral lending relationships.

Finally, we note also that the model does a poorer job in explaining the observed average interest rate level. This partly reflects the fact that the model abstracts from time-varying bank specific outside options for lending and borrowing that are associated to massive central bank interventions during the financial crisis, in particular after the Lehman failure. Further, the skewness of the cross-sectional interest rate distribution is over-estimated and there are too many reciprocal links compared to the observed network. Note also that the model abstracts from any bank heterogeneity beyond differences in liquidity shocks, in particular differences in balance sheet strength or heterogeneous outside options.

4 Model Analysis

One of the most appealing features of structural models is the possibility to answer meaningful economic questions and derive relevant policy implications. As we shall now see, the estimated structural model described above can be used not only to extract relevant information, but also, as an interesting testing ground for different policies affecting the interbank lending market. Here we focus on assessing the role of private information, gathered through monitoring and repeated interaction, in reducing asymmetric information problems about counterparty risk and fostering the efficiency of the interbank market.

4.1 Simulated Paths

Figure 2 below shows a simulated path covering roughly 4 weeks of operations in the interbank market under the estimated structural parameters. In particular, this figure plots the paths of daily mean log traded volume and daily mean density, stability, reciprocity, mean monitoring, mean search, perceived error variance (PEV) and link probabilities. Figure 2 shows also the actual network connections established at three different moments of time labeled T_1, T_2 and T_3.

The top graph in Figure 2 conveniently shows how daily network density (solid line) and daily mean log traded volumes (dashed line) are negatively related over time. This negative relation is justified by the fact that periods of low density are characterized by the inactivity of smaller banks in the interbank market. The banks that remain active are large banks whose trade volume is typically large, and hence, average trade volumes rise

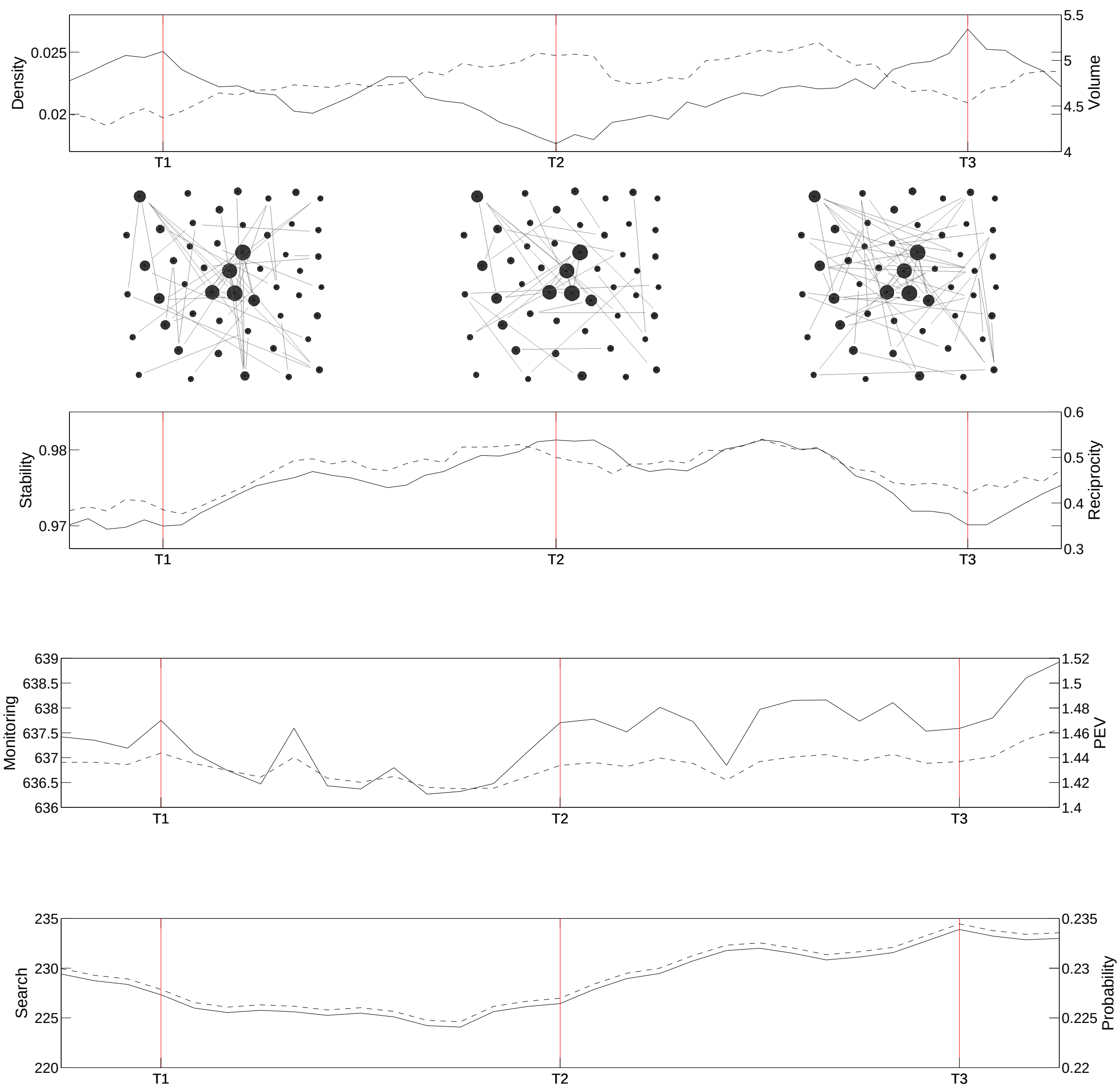


Figure 2: Simulated 5-week interbank network path and network plots at given moments of time T_1, T_2 and T_3. Larger nodes reflect larger banks as measured by total lending and borrowing volume. Left axes correspond to solid lines, right axis to dashed lines. Mean volumes of granted loans are reported in log million euro, mean monitoring and search expenditures in thousands of euro.

in crisis periods. From a causal perspective, a larger volume of trade in a given period will make banks update positively their expectations about future traded volumes, and consequently update positively their expectation on future profit opportunities. This, in turn, will give an incentive to increase monitoring and search which leads ultimately to a higher number of trades and higher network density. However, this causal relation is played out over a number of days and several new shocks will occur in the meanwhile. As a result, at a daily frequency, the negative contemporaneous relation between density and average traded volumes remains.

The network plots in Figure 2 are essentially 'snapshots' of the network connections on the days marked T_1, T_2 and T_3. These plots show how the large banks (as measured by total lending trading), identified here by the larger nodes, are more active on average (i.e. have more connections with other banks). These banks have more to gain from interbank market operations and hence monitor and search more. On the contrary, the smaller banks, depicted in the periphery with smaller nodes, will participate in the market only sporadically. This is especially true in periods of low activity like T_2 when only the larger banks remain active.

Figure 2 shows also how daily stability (solid line) and daily reciprocity (dashed line) are positively correlated. This relation is justified by the fact that both stability and reciprocity are negatively correlated with density, see also the observed and simulated auxiliary statistics. This negative relation emerges essentially once again because periods of low density are characterized by the inactivity of smaller banks while the large banks remain active. Since larger banks tend to create more stable and reciprocal relations (where they act as both lenders and borrowers), the stability and reciprocity rises. In essence, in periods of low activity in the interbank market like T_2, where only the large banks with very strong and reciprocal relationships are active, the daily stability and reciprocity is large. On the contrary, in periods of high activity like T_3, smaller banks that infrequently access the market as either lenders or borrowers become active and hence the network stability and reciprocity becomes low.

Finally, Figure 2 documents also how monitoring and search (solid lines) and the perception error variance (PEV) and the link probability (dashed lines) evolve over time. Inspection of the plotted path for monitoring and PEV reveals that PEV plays a major role in determining the optimal monitoring decision. The role of PEV on monitoring decisions is described by the derived Euler equation for monitoring. It is also clear from the graphs how both monitoring and search are positively related to variations in density. Indeed, the rise in density influences positively the expected profitability of the interbank market and hence leads to increase in search and monitoring. In turn, the rise in search and monitoring causes an increase in trade intensity that feeds back into the system. Similarly, a fall in trade density can have prolonged effects due to this feedback mechanism. Ultimately, this can lead to a halt in the market. This is clear if one considers, for example, the decline in density monitoring and search from T_1 to T_2 and the rise in all three variables until T_3.

The events described here are short lived and occur at a high frequency. However, as we shall see below, depending on the action of central bank authorities, reactions of a much larger magnitude occurring at a much lower frequency can essentially bring the interbank market into a halt for extended periods of time.

4.2 Network Uncertainty: Unconditional Network Properties and Optimal Search and Monitoring

Figure 3 characterizes the simulated network by describing the distribution of mean densities, interest rates and traded volumes of the top 50 banks operating in the Netherlands over various possible network structures. In particular, these figures were obtained by first drawing the bank properties (as described by the parameters μ_μ, σ_μ, μ_σ and σ_σ) and then calculating the unconditional mean of densities, interest and volumes both over all banks and over $T = 150$ time periods. This procedure was then repeated in a Monte Carlo setting with 500 replications. Hence, the uncertainty in Figure 3 reflects essentially the uncertainty about the exact network structure as described by the unobserved liquidity shock distribution.

Figure 3 shows that mean log traded volumes can vary considerably depending on the network structure. In particular, the inter-quartile includes the observed average log volume of 4.12 reported in Table 6 but ranges from slightly below 2.5 to above 6.0, depending on the precise structure of the network. Furthermore, the relative symmetry of mean log volumes reveals that traded volumes in levels are highly skewed across networks. This result is justified by the fact that the mean traded volumes depend considerably on the presence of large complementary banks whose shocks are large in magnitude and negatively correlated. The presence of two large banks with large liquidity shocks that feature, on average, liquidity shocks with opposite signs will foster a very efficient usage

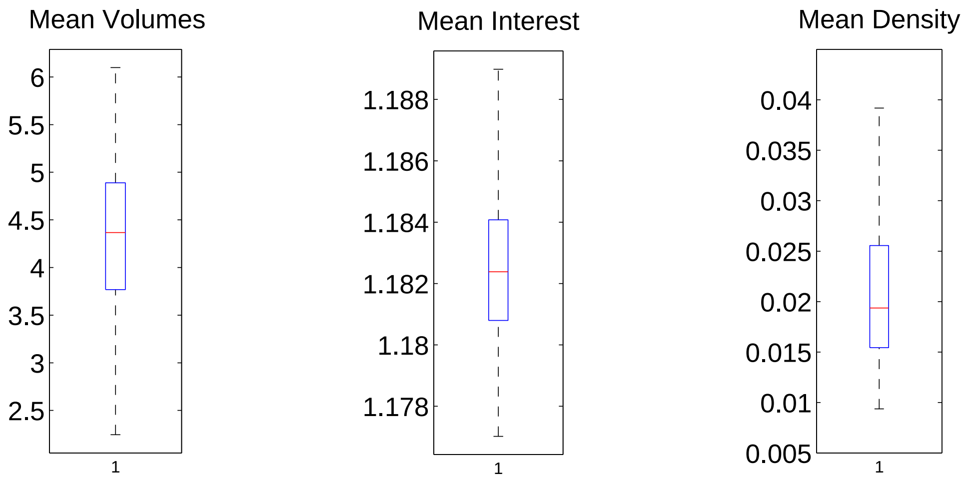


Figure 3: Box plot featuring, median (red line), inter-quantile range (box) and outlier bound (whiskers) of mean log volume, mean interest rate, and mean density with approximately 99% coverage; see McGill et at. (1978).

of the interbank market to smooth shocks as these banks will monitor each other closely and obtain very favorable interest rates.

Figure 3 also shows that the mean density of the network, which characterizes the average number of interbank market operations per day, depends strongly on the network structure. Once again, the inter-quantile range includes the observed density of 0.0212 reported in Table 6 and the large variance is justified by the fact that interbank market trade is intensified when banks complement each other well and build relations over time. The interbank market trade is appealing when banks monitor each other and hence obtain favorable interest rates (relative to the central bank bounds). However, intense monitoring will only occur when the banks expect significant profit opportunities in the interbank market. These profit opportunities increase with the size of the liquidity shocks and the possibility of finding partners facing a symmetric liquidity restriction. As such, monitoring will be increased in networks featuring banks that face large and/or symmetric liquidity shocks.

Contrary to the density and volume, Figure 3 shows that the unconditional mean of interest rates of granted loans is rather stable across different networks. This means that, for given central bank bounds, different networks will produce largely different outcomes in terms of mean trade intensity and mean volume, but very similar outcomes in terms of mean interest rates. In essence, the decision to trade or not is very sensitive to the characteristics of the partners, but the conditions under which trade occurs do on average not vary much across different network structures.

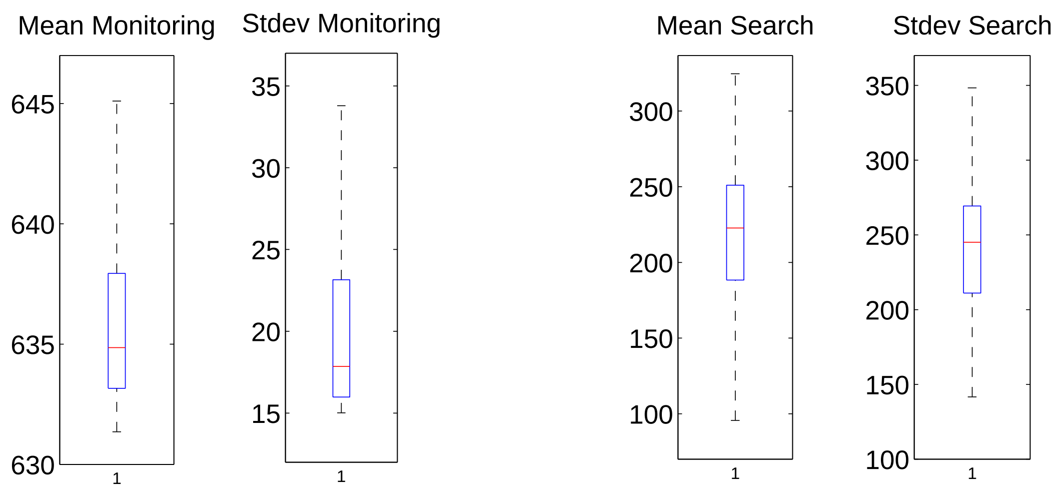


Figure 4: Box plot featuring, median (red line), inter-quantile range (box) and outlier bound (whiskers) of mean and standard deviation of monitoring and search with approximately 99% coverage; see McGill et at. (1978).

Figure 4 shows the implied optimal mean and standard deviation in search and monitoring expenditures (annualized expenditures per bank, in thousands of euro) of the top 50 banks operating in the interbank market. As expected, the mean monitoring value is highly skewed and varies considerably across network structures. Both the large skewness and large variance are in accordance with the analysis made above that argued how different network structures led to different monitoring efforts, and ultimately, to different trade intensities and lending volumes in the interbank market. The same type of argument is valid for the wide variation in search effort that doubles within the 99% confidence bound. Indeed, since trade opportunities depend on search (which determines the probability of an open link), the large variation in search is consistent with the observed large variation in density across network structures. Moreover, our model formulation allows a boundary solution with zero search expenditures.

Economically, the estimated median of monitoring expenditure is about 635 thousand euro per bank (annualized), the median of search expenditures about 240 thousand euro. Note that these figures are for the average bank in the Dutch market, and they are higher for more active banks. While our model indicates that the cross-sectional standard deviation of monitoring is relatively low, the cross-sectional standard deviation of search expenditures is considerably larger. An observation that is also driven by the fact that our model allows a boundary solution with zero search expenditures. This leads, depending on the network configurations, to the large variance in search.

4.3 Central Bank Policy Analysis: The Interest Corridor Width Multiplier

Finally, we analyze how changes in the width of the central bank's interest corridor affect the market by changing the outside options for lending and borrowing. In particular, we show how even a small change in the interest bounds can have a strong effect on the overall interbank market activity. Specifically, we show that due to a reduction in search and monitoring, the decrease in market activity is a multiple of the decrease in the bounds. In this sense, we uncover a multiplier for the interbank market that is important for policy makers. This multiplier increases with the level of monitoring and search.

Figure 5 shows how the width of the EBC corridor produces significant changes in the mean network density. Again, the uncertainty captured by the quantiles in the box plot is associated to a larger extent to the uncertainty about the precise network structure.

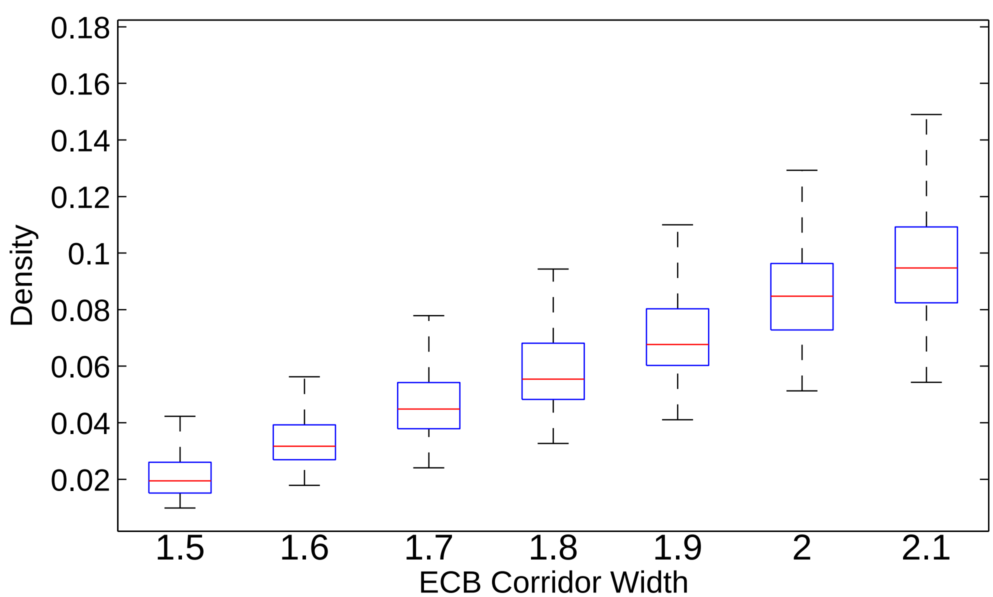


Figure 5: Box plot featuring, median (red line), inter-quantile range (box) and outlier bound (whiskers) of mean network density for different ECB corridor sizes. Each box plot has approximately 99% coverage; see McGill et at. (1978).

The most striking feature of Figure 5 is that an increase of roughly 50% in the width of the ECB's interest rate corridor (from 1.5 to 2.1) is predicted to produce a 4 fold increase in the median network density (average number of daily trades) from roughly 2% density to over 8%. Furthermore, at a corridor width of 2%, the lower bound of the 99% confidence interval across all network structures is larger than the upper bound on the 99% confidence interval across network structures at a corridor width of 1.5%. This analysis shows that these effects are highly significant and that the ECB corridor width plays an important role in the intensity of interbank activity.

A second important feature of Figure 5 is that, since the model is nonlinear, the multiplier value is not constant over the range of ECB corridor widths. In particular, Figure 5 shows that the multiplier value is decreasing with corridor width. Indeed, a 0.1 unit increase on the bound has a much larger relative effect in density for low corridor widths compared to large ones.

The presence of this multiplier, as well as its nonlinearity, are both explained by the role that monitoring and search play in the interbank market. Just like in the usual Keynesian multiplier, the effects of a change in the width of the ECB bounds can also be decomposed into *(i)* an immediate short-run effect, and *(ii)* a long-run effect that is created by the multiplier that results from feedback loops between the effect of monitoring and search on loan outcomes and expectations about credit conditions.

Consider a decrease in the width of the ECB corridor. In the advent of such a shock, the interbank market will immediately shrink as a fraction of lending and borrowing operations are no longer profitable given the new tighter ECB bounds. In this immediate 'mechanical' effect, part of the interbank market is simply 'substituted' by lending and borrowing with the central bank authority which now plays a more important role. This immediate short-run effect is however only a fraction of the total long-run effect. Indeed, given that the possibilities of trade are now smaller, the expected future profit is diminished and the incentive to monitor and search partners is reduced. This reduction is monitoring and search (depicted in Figures 6 and 7) will further reduce the mean density and mean traded volumes in the interbank market. These, in turn force banks to revise the expected profitability of monitoring and search efforts, pulling these variables further down. This 'negative spiral' that defines the multiplier will bring the market to a new level of operation that can be orders of magnitude lower than that observed prior to the bounds.

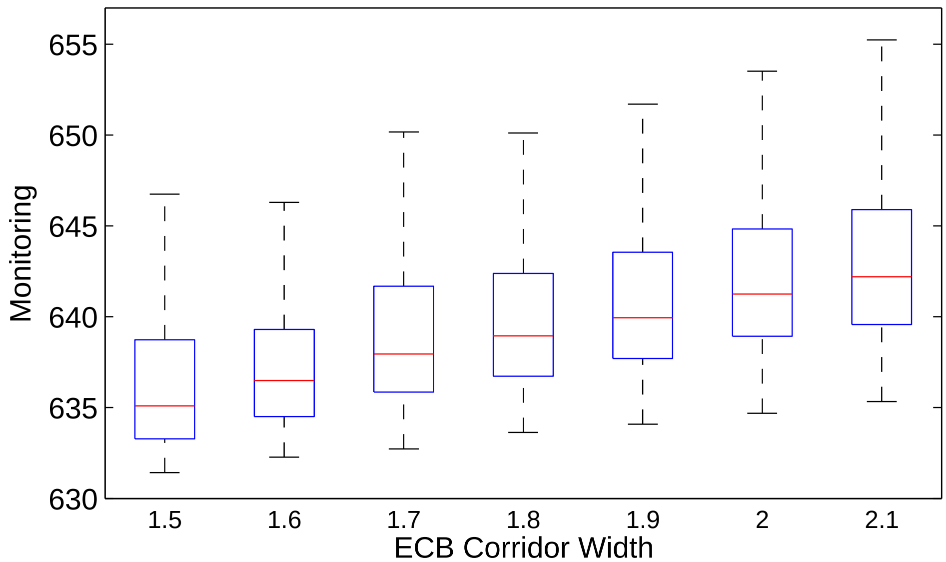


Figure 6: Box plot featuring, median (red line), inter-quantile range (box) and outlier bound (whiskers) of mean monitoring per bank over alternative ECB interest corridor width (annualized and in thousands of euro). Each box plot has approximately 99% coverage; see McGill et at. (1978).

Hence, if the ECB wishes to get tighter control over the traded rates by narrowing the corridor it has to expect further adverse effects on interbank lending activity triggered by a reduction of monitoring and search. On the other hand, if the ECB wants to foster an active decentralized interbank lending market as a means to explore benefits from peer monitoring it is essential to consider policies that increase the difference between the standing facilities for depositing and lending funds. Only thereby the interbank market is profitable enough to encourage intense peer monitoring and search among banks. In both cases the multiplier effect should be taken into account when considering policy changes.

5 Conclusion

In this paper we propose a structural micro-founded network model for the unsecured interbank lending market. We use this model to study asymmetric information problems about counterparty risk and search frictions, and the interaction of both. Specifically banks are profit maximizing agents that can engage in costly peer monitoring to mitigate

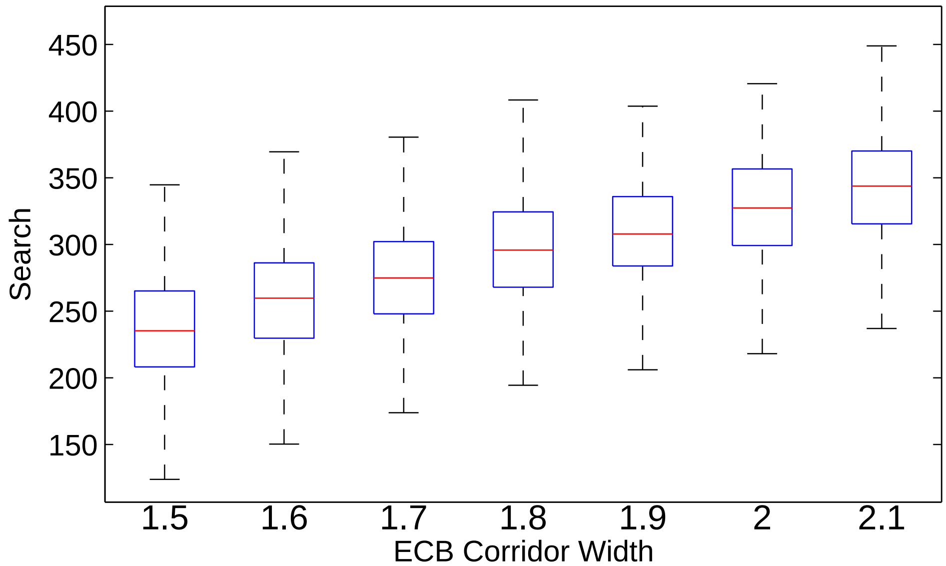


Figure 7: Box plot featuring, median (red line), inter-quantile range (box) and outlier bound (whiskers) of mean search per bank over alternative ECB interest corridor width (annualized and in thousands of euro). Each box plot has approximately 99% coverage; see McGill et at. (1978).

the counterparty risk uncertainty and search for preferred trading opportunities. We provided a solution to the banks' dynamic optimization problems of allocating search and monitor across potential trading partners. The solution crucially depends on expectations about future trading opportunities in the network.

In a second part of the econometric analysis we estimate the structural model parameters using loan level data at daily frequency collected for the Dutch unsecured overnight interbank lending market from 2008 to 2011. The parameter vector is estimated with the method of indirect inference that builds upon a set of auxiliary statistics that capture the dynamic structure of the observed interbank network. The estimated parameter vector shows that asymmetric information problems, interacted with monitoring and search efforts, have significant effects on the network structure and credit conditions. In particular we find that (i) monitoring has a significant effect on the reduction of credit risk uncertainty, (ii) monitoring depends positively on the expected future loan probability and volume, (iii) bilateral lending relationships with increased lending activity and lower interest rates emerge, similar to those documented in the observed Dutch data, (iv) the fundamental driving force of trading opportunities are significant differences in the structure of liquidity shocks across banks.

Finally, we use the estimated model to discuss several policy implications. In particular, we show that in order to foster the trading activity in unsecured interbank markets and associated benefits from peer monitoring an effective policy measure is to widen the bounds of the interest corridor. The effects of a wider corridor result from both a direct effect and a non-linear, indirect multiplier effect triggered through increased monitoring and search activity among banks.

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A Derivatives in FOC

In the first-order conditions that characterize any interior solution with positive monitoring and search expenditures, we have $\frac{\partial \pi_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = \frac{\partial \bar{R}_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} y_{i,j,t} + \bar{R}_{i,j,t} \frac{\partial y_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}}$ and $\frac{\partial \pi_{i,j,t}}{\partial s_{i,j,t}} = (r - r_{j,i,t}) \frac{\partial y_{j,i,t}}{\partial s_{i,j,t}}$. These derivatives can further be unfolded by the chain rule and the following partial derivatives

$$
\begin{aligned}
&\frac{\partial \phi_{i,j,t}}{\partial m_{i,j,t}} = \beta_\phi, \quad \frac{\partial \xi_{i,j,t}}{\partial \phi_{i,j,t}} = \frac{-\gamma_\sigma \tilde{\sigma}^2_{i,j,t} \exp(\beta_\sigma \phi_{i,j,t}) \beta_\sigma}{(1+\exp(\beta_\sigma \phi_{i,j,t}))^2}, \quad \frac{\partial P_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = \frac{\epsilon^2}{(\sigma^2_{j,t} + \tilde{\sigma}^2_{i,j,t} + \epsilon^2)^2} \\
&\frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = 0.5/\epsilon^2, \quad \frac{\partial \tilde{R}_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = -\frac{\partial P_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} + \frac{\partial 1 - P_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} r_{i,j,t} + (1 - P_{i,j,t}) \frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} \\
&\frac{\partial y_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} = C'_{i,j,t} \left(\frac{-\beta_I \exp(-\beta_I (r_{i,j,t} - \frac{P_{i,j,t}}{1-P_{i,j,t}}))}{1+\exp(-\beta_I (r_{i,j,t} - \frac{P_{i,j,t}}{1-P_{i,j,t}}))} \cdot \left(\frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} - \frac{\partial P_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}} (1 - P_{i,j,t})^{-2} \right) \right. \\
&\qquad\qquad \left. + \frac{\beta_I \exp(-\beta_I (r - r_{i,j,t}))}{1 + \exp(-\beta_I (r - r_{i,j,t}))} \left(-\frac{\partial r_{i,j,t}}{\partial \tilde{\sigma}^2_{i,j,t}}\right) \right) I_{i,j,t}, \\
&\frac{\partial y_{j,i,t}}{\partial s_{i,j,t}} = C''_{i,j,t} \frac{\beta_\lambda \exp(-\beta_\lambda (s_{i,j,t} - \alpha_\lambda))}{(1 + \exp(-\beta_\lambda (s_{i,j,t} - \alpha_\lambda)))^2}
\end{aligned}
$$

where $C'_{i,j,t} := \eta_{i,j,t} \zeta_{i,j,t}$ does not depend on monitoring and $C''_{i,j,t} := U_{i,j,t} I_{i,j,t} \zeta_{i,j,t}$ does not depend on the search efforts $s_{i,j,t}$.

B Network Auxiliary Statistics

In this section we provide formulas for the non-standard auxiliary statistics that characterize specifically the network structure of the interbank lending market.

$$
\begin{aligned}
density_t &= \frac{1}{N(N-1)} \sum_{i,j} l_{i,j,t}, \quad reciprocity_t = \frac{\sum_{i,j} l_{i,j,t} l_{j,i,t}}{\sum_{i,j} l_{i,j,t}}, \\
stability_t &= \frac{\sum_{i,j} \left(l_{i,j,t} l_{i,j,t-1} + (1 - l_{i,j,t})(1 - l_{i,j,t-1}) \right)}{N(N-1)}.
\end{aligned}
$$

The interbank network is also characterized by its *degree distribution*. The degree centrality of a bank counts the number of different trading partners. For directed networks the out- and in-degree of node i are given by

$$
d_{i,t}^{out} = \sum_j l_{i,j,t} \quad \text{and} \quad d_{i,t}^{in} = \sum_j l_{j,i,t}.
$$

Instead of considering all $2N$ variables individually, we consider the mean, variance and skewness of the out-degree distribution. Note that the mean of degree distribution is proportional to the density.

The (local) clustering coefficient of node i in a binary unweighted network is given by

$$
c_{i,t} = \frac{1/2 \sum_j \sum_h (l_{i,j,t} + l_{j,i,t})(l_{i,h,t} + l_{h,i,t})(l_{j,h,t} + l_{h,j,t})}{d_{i,t}^{tot}(d_{i,t}^{tot} - 1) - 2 d_{i,t}^{\leftrightarrow}},
$$

where $d_{i,t}^{tot} = d_{i,t}^{in} + d_{i,t}^{out}$ is the total degree and $d_{i,t}^{\leftrightarrow} = \sum_{j \neq i} l_{i,j,t} l_{j,i,t}$, see Fagiolo (2007) for details. We again maintain consider the mean and standard deviation of the local clustering coefficients.

Second, we compute simple bilateral local network statistics that measure the intensity of a bilateral trading relationship based on a rolling window of size T_{rw}. As a simple measure of bilateral relationships, we compute the number of loans given from bank i to bank j during periods $t' = \{t - T_{rw} + 1, ..., t\}$ and denote this variable by

$$
l_{i,j,t}^{rw} = \sum_{t'} l_{i,j,t'},
$$

where the sum runs over $t' = \{t - T_{rw} + 1, ..., t\}$. Further, for any bank pair we compute the lender and borrower preference index during the rolling window

$$LPI_{i,j,t} = \frac{y_{i,j,t'}}{\sum_{j,t'} y_{i,j,t'}} \text{ and } BPI_{i,j,t} = \frac{y_{i,j,t'}}{\sum_{i,t'} y_{i,j,t'}}$$

that measure the relative importance of banks forming a pair in terms of portfolio concentration.

C Summary statistics of the data

Table 7: Descriptive Statistics of Dutch Interbank Network: The table shows moment statistics for different sequences of network statistics and cross-sectional correlations that characterize the sequence of observed Dutch unsecured interbank lending networks. The statistics are computed on a sample of daily frequency from 1 February 2008 to 30 April 2011.

Statistic	Mean	Std	Autocorr	Skew	Kurtosis
Density	0.0212	0.0068	0.8174	0.8667	3.1983
Reciprocity	0.0819	0.0495	0.2573	0.2903	2.8022
Stability	0.9818	0.0065	0.8309	-0.8590	3.0503
Mean-out-/indegree	1.0380	0.3323	0.8174	0.8667	3.1983
Std-outdegree	1.8406	0.4418	0.6882	0.0553	2.4326
Skew-outdegree	2.8821	1.0346	0.7035	0.6074	2.4572
Mean-indegree	1.0380	0.3323	0.8174	0.8667	3.1983
Std-indegree	1.6001	0.4140	0.6880	0.6997	3.4529
Skew-indegree	2.4030	0.8787	0.6576	0.6714	2.7434
Outdegree-corr	0.0000	0.0000	-0.0225	-0.2675	5.6217
Indegree-corr	0.0000	0.0000	-0.0604	0.1348	4.6186
Mean-clustering	0.0308	0.0225	0.4149	0.7900	3.2473
Std-clustering	0.0880	0.0490	0.3587	0.1561	2.7280
Skew-clustering	3.7367	1.5454	0.1213	-0.2213	3.1281
Avg log-volume	4.1173	0.2818	0.4926	-0.2820	2.8220
Std log-volume	1.6896	0.1685	0.3623	0.1541	3.4546
Skew log-volume	-0.3563	0.2818	0.2970	-0.0669	3.2151
Avg rate	0.2860	0.3741	0.9655	1.1044	2.6965
Std rate	0.1066	0.0632	0.7865	1.6668	6.8848
Skew rate	0.6978	1.6399	0.5492	0.6832	2.9469
Corr(r,l^{rw})	-0.0716	0.1573	0.4066	0.0817	2.8539
Corr(l,l^{rw})	0.6439	0.0755	0.4287	-0.7653	4.2833
Corr(l,LPI^{rw})	0.4085	0.0749	0.4365	-0.1585	3.8179
Corr(l,BPI^{rw})	0.4379	0.0730	0.4604	-0.3398	3.4158

D Indirect Inference Weights

Following Gourieroux et al. (1993), we adopt a quadratic objective function for indirect inference estimation with diagonal weight matrix based on the optimal weighing matrix that is obtained as the inverse of the covariance matrix of the auxiliary statistics. In particular, we set the criterion function to

$$\hat{\boldsymbol{\theta}}_T := \arg\max_{\boldsymbol{\theta}\in\Theta} \left[\hat{\boldsymbol{\beta}}_T - \frac{1}{S}\sum_{s=1}^{S}\tilde{\boldsymbol{\beta}}_{T,s}(\boldsymbol{\theta})\right]\mathbf{W}_T\left[\hat{\boldsymbol{\beta}}_T - \frac{1}{S}\sum_{s=1}^{S}\tilde{\boldsymbol{\beta}}_{T,s}(\boldsymbol{\theta})\right]',$$

but we use a weight matrix $\mathbf{W}_T$ that differs from the optimal for several reasons. First, the inverse of the covariance matrix is only optimal under an axiom of correct specification. Moreover, even under correct specification the (asymptotically) optimal weighing matrix can lead to larger variance of the estimator in finite samples. Second, for economic theoretic reasons, there are a number of auxiliary statistics that we wish to approximate better than others.

As such we adopt a matrix $\mathbf{W}_T$ corresponding to a *tanh* transformation of the the diagonal of the inverse of the covariance matrix of the auxiliary statistics. Further the weight of the density and corr$(l_{i,j,t}, l^{rw}_{i,j,t-1})$ is increased by 10 and the weight of corr$(r_{i,j,t}, l^{rw}_{i,j,t-1})$ is increased by 500 as we want to match these characteristics particularly well. For computational reasons we minimize the logarithmic transformation of the objective function. For each of the $S = 12$ network paths we burn 1000 periods to minimize dependence on the initial value. The numerical optimization of the objective function is done using a grid search algorithm.