12 What is the topology of your payment network? Node-degree distributions

Payment networks have a structure that can be analyzed and described using tools from graph theory. To do this, we take the banks to be the nodes of the network. If two banks exchange payments they share a link. This allows us to apply many insights from other networks to banking.

The number of links is also called the node-degree of a node. The statistical distribution of this node-degree reveals a lot about the nature of the network. If links are formed randomly between nodes, the node-degree distribution follows a rather “tame” distribution: most nodes will have 1 or 2 links while there may be a few with as much as 10 or 12. Analysis of real-world networks like the Internet, however, reveals that most nodes indeed have a few links, but a few nodes have 10,000 or even 1 million links. Such nodes with a high node-degree act as the hubs of the network. The node-degree in these real-world networks appears to follow a Power-law distribution.

Research has found such structures in networks as diverse as the pages of the world-wide-web, board directors of fortune 1000 companies, and the spread of AIDS. [1]

Networks with such ‘megahubs’ are formed through a process of preferential attachment of new nodes: links do not form randomly, but instead a new node is much more likely to link itself to an existing node that already has a high number of links.

[1] Barabási (2003). For the www, the nodes are pages and the links are the hyperlinks to other pages. For company directors the directors are the nodes, 2 directors are said to be linked if they serve on the same board. For the spread of AIDS, the nodes are infected individuals who share a link if one has infected the other.

Analysis of payment networks also reveals such long tails and megahubs. For example Figure 1 shows the node-degree distribution for the banks on the SWIFT network.

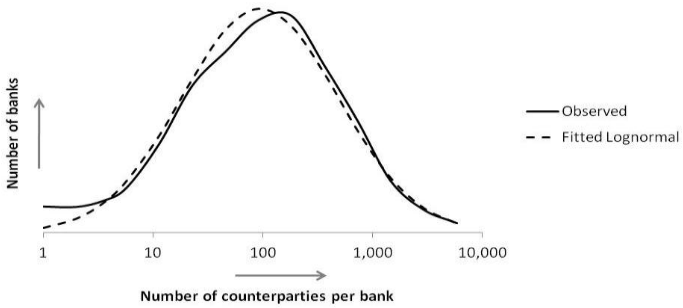


Figure 1: node-degree distribution of the SWIFT network

The actual curve is close to a normal distribution, suggesting that the underlying variable is Log-normally distributed. The dotted line plots a fitted curve.[2] Figure 2 shows the corresponding log-log frequency plot.

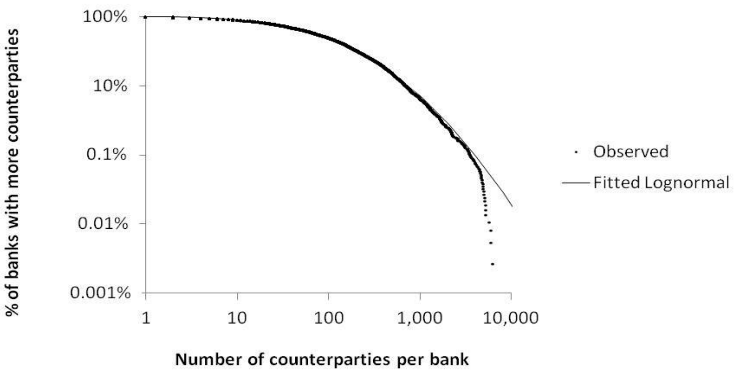


Figure 2: Log-log frequency plot of node-degree on SWIFT network

[2] With parameters $\hat{\mu}$ =4.9 and $\hat{\sigma}$ =1.7. As before these were calculated using the Maximum Likelihood Estimators.

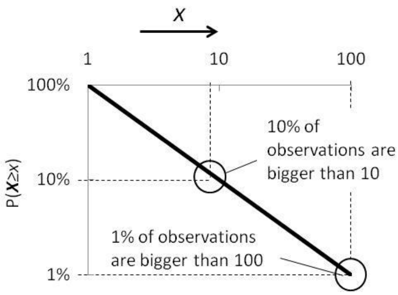
Here we find that for the upper tail the distribution falls below the Log-normal distribution. One explanation could be that the total number of Banks on the SWIFT network puts an upper limit on the number of links for any node.

Power-law distributions and log-log frequency plots

A Power-law is a type of relationship between the size events and their frequency. Consider the strength of earthquakes as measured on the Richter scale (where a force 7 quake is 10 times as strong as a force 6 event). In Southern California earthquakes exceeding force 5 on the Richter scale happen about once a year while earthquakes exceeding force 6 happen every 10 years, and earthquakes exceeding force 7 occur only once in 100 years.

Formally, a Power-law states that $P(X \geq x) = x^{-\alpha}$, where $P(X \geq x)$ denotes the probability that some variable X is bigger than a specific value x. The Power-law distribution has a key parameter α which happens to be equal to 1 in the case of earthquakes.

A key feature of Power-law distributions is that the log-log plot shows a straight line, where the slope of the line is equal to the parameter α. The plot below shows this for a Power-law distribution with α=1.



The sizes of human settlements, the intensity of wars, the size of meteorites, income and wealth, the size of files sent over the Internet and natural phenomena such as rainfall, hurricanes and earthquakes all appear to follow a Power-law.

Note that Figure 2 follows a straight line for the middle part. Several other studies of other payments networks, such as Fedwire, CHAPS and BOJ-net, have found the node-degree distribution to follow power laws. Table 1 summarizes the results.

Table 1: Node-degree distributions of selected payment networks[3]

Network	**Size (# nodes)**	**Average node-degree**	**Distribution**
SWIFT	9,000	295	*Log-normal* (μ=4.9; σ=1.7)
Fedwire	6,660	15.2	Power-law (α=1.1)
CHAPS	337	2.9	NA
BOJ-net			Power-law (α=1.3)

These findings are relevant, because scale–free networks are different from random networks in several aspects. First, the average path-length is shorter. A path is a set of (directed) links that allows one to go from one node to another. Consider the extreme case of a hub and spoke network: one megahub in the middle connected to all other nodes: here it takes at most 2 steps to go from any node to any other. In a random (Erdõs, Rényi) network the average path can be much longer. The megahubs effectively serve as conduits to shorten the paths.

Second, much analysis has been done on the vulnerability of networks to (systemic) failures. Typically, scale free networks are robust to random failures, because most nodes have few links. If nodes fail at random, the probability that a systemically important hub fails is quite low. By contrast such networks are very vulnerable to attacks that target the megahubs.

[3] Figures for Fedwire from Soramäki, Bech (2006), Chaps from Beckhr, Millard (2008), BOJ-net from Inaoka, Ninomaya (2004).