11 Does your amount start with a 1? Benford's law

If numbers are more or less evenly spread over multiple orders of magnitude, Benford's law applies: the first digit of the amount is much more likely to be 1 than 9. Benford law's predicts that for 30% of all payments the first digit of the amount is a 1, while it is a 9 for only 5% of payments.

Figure 1 below shows the distribution by leading digit for a sample of SWIFT transfers, which is remarkably close to Benford's law. Perhaps the only noteworthy deviation is the relatively high frequency of the digit 5. One explanation could be that the limit for free of charge SEPA transfers is 50,000 Euro.

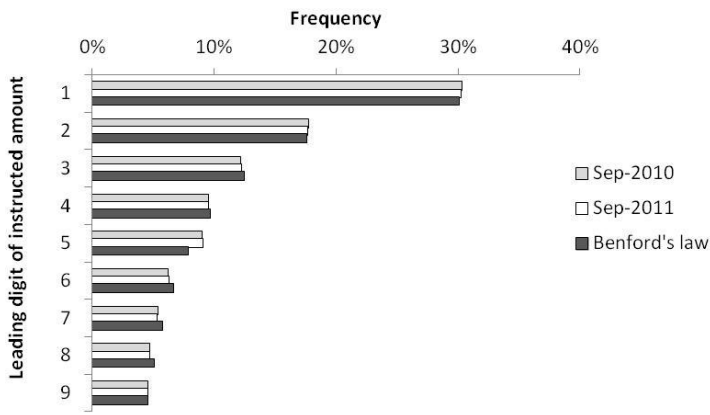
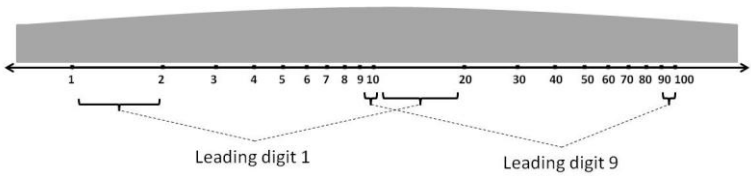


Figure 1: Benford's law compared to SWIFT transfers

Benford's law applies to any set of numbers that spans several orders of magnitude such as incomes on tax returns, annual sales of firms, and the population of cities. An interesting, and often quoted, application of Benford's law is fraud prevention, where allegedly cooked books were be spotted because fraudsters used a random generator to for each digit, putting all digits with equal frequency in the lead position.

Benford's law

To see why the digit 1 is more likely, consider the below scale: if numbers are equally likely to be between 1 and 10 as they are to be between 10 and 100, a log scale would be the appropriate one to use. On this scale, numbers starting with 1 occupy much more space than higher digits. So it seems logical that the leading digit 1 occurs most often.



The mathematics follow quite easily from this log scale: let p_n be the probability that the first digit is n, then according to Benford's law:

$$p_n = \log_{10}(n+1) - \log_{10} n = \log_{10}\left(1 + \frac{1}{n}\right).$$

This works out to about 30% for the digit 1 and only 5% for the digit 9, quite a difference.