10 How big is that payment? The frequency distribution of payments by size

Chapter 1 analyzed the size of cash payments. We now do the same for interbank transactions over networks such as SWIFT and Fedwire. These are several orders of magnitude larger than cash transactions: the average SWIFT payment is the equivalent of 400,000 euro and the average Fedwire payment is even larger: 1,200,000 dollar. Interestingly their size follows the same type of statistical distribution as cash payments, namely the Log-normal distribution described in chapter 1. Figure 1 shows a histogram for all payments made over the SWIFT network during Oct 2010, using the logarithm of the transaction size.

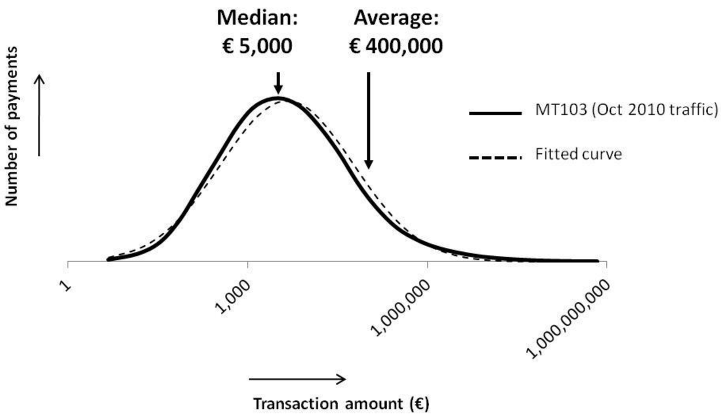

Figure 1: Distribution of SWIFT transfer instructions by amount

The figure also shows a fitted Log-normal curve.[1] The actual observed curve has a thinner left-hand tail and a thicker right-hand tail than the fitted Log-normal curve.[2]

---

[1] With Maximum Likelihood Estimate (MLE) parameters $\hat{\mu}$ =8.4 and $\hat{\sigma}$ =2.5.

We saw in chapter 1 that small cash payments follow this same statistical distribution, albeit with different parameters. There is evidence that the sizes of other payment mechanisms also follow a Log-normal distribution. Table 1 below gives an overview:

Table 1: frequency distribution by size of selected payment instruments[3]

Instrument	Average value	Median value	$\hat{\mu}$	$\hat{\sigma}$
Cash	25	15	2.7	1.1
Debit cards	65	43	3.8	0.9
SWIFT	400,000	5,000	8.4	2.5
T2	1,250,000	20,000	9.9	2.9
Fedwire	3,000,000	30,000	10.3	3.7

The obvious question is: why? What process would generate this apparently pervasive distribution of payment size? The honest answer is: we don't know. We know several processes that ultimately lead to a Log-normal distribution.

The central limit theorem states that the sum of a large enough number of variables, each with an identical distribution, will eventually approach a Normal distribution. The interesting part is that the variables can have any distribution (as long as they are identical and independent of each other) yet still their sum will (eventually) follow a Normal distribution. Similarly, the product of a large number of variables, each with an identical distribution, will eventually be Log-normally distributed.

---

[2] This is in line with the fact that the observed average (about EUR 400,000) is higher than the theoretical average which is equal to $e^{\hat{\mu}+½\hat{\sigma}^2} = 101{,}215$.

[3] Figures for Cash from Boeschoten, Fase (1989), De Grauwe, Buyst 2000) and Kippers (2004), Debit Card figures from BIS, T2 from Rosati, Secola (2005) and Fedwire from Soramäki, Bech (2006). All values in euro, except Fedwire, which are in US dollars.

We can speculate about which multiplicative process drives payment size. It remains striking that the same distribution applies across a wide range of payment instrument and payment sizes.

Interpreting the parameters μ and σ

In the Normal distribution μ is the median (middle) observation and σ denotes confidence intervals: 2/3rds of observations fall in the interval (μ-σ, μ+σ), a range with width 2σ around the middle. For the Log-normal distribution the median is equal to e^{μ}, where *e* is Euler's constant described in chapter 6. Parameter σ still gives a sense of spread but differently. 2/3rds of observations fall between the median *multiplied* by e^{σ} and the median *divided* by e^{σ}.

For cash payments, the median is 15 euro, while $e^{\sigma} = e^{1.1}$ ≈3. So 2/3rds of observations fall into the interval from 15/3 to 15*3. This range covers a factor 9, or close to one order of magnitude (a factor 10). For the SWIFT transactions we get $e^{2.5}$ ≈12 and the interval covers a factor 144 or slightly over 2 orders of magnitude. In fact a good rule of thumb is that 75% of observations fall in an interval that covers σ orders of magnitude.

80/20 and beyond

The economist Vifredo Pareto observed that US incomes follow a Power-law with α around 1.5. The number of people earning more than 20,000 is 30 times the number earning more than 200,000 and 1000 times the number of people earning 2 million or more.[4]

The "80/20 rule" is also called the Pareto principle, after his observation that 20% of the people own 80% of the land. There is a relation with the parameter α: for α=1.5 we get "70/30": 30% of people earn 70% of income, and the top 1% earns slightly over 22%. It takes α=1.16 to get to 80/20, where the top 1% has 50%.

[4] since α=1.5, if the probability decreases by a factor 10 then the income grows by $10^{1.5}\approx 30$, and if it decreases by a factor 100 then income grows by $100^{1.5}=1000$