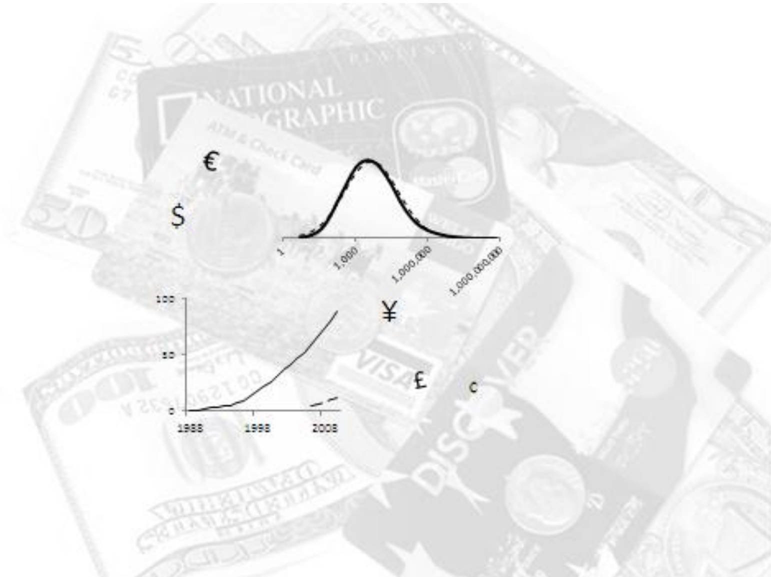
SWIFT

A BILLION HERE, A BILLION THERE: THE STATISTICS OF PAYMENTS



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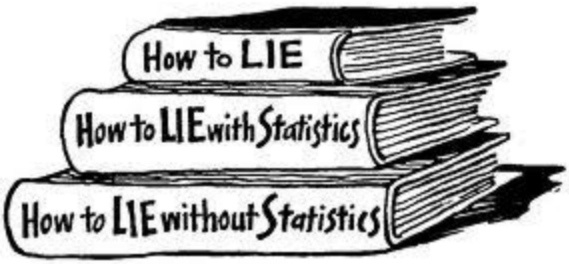

How to LIE
How to LIE with Statistics
How to LIE without Statistics

Introduction

Payments are part of everyday life as well as a crucial element of the financial fabric of society. This book focuses on the statistical aspects of payments in two different ways. It looks at payment statistics: the number and size of transactions as well as the type of instruments we use. But it also applies tools from mathematical branch known as statistics to these figures: frequency distributions, growth curves and relationships between variables.

The focus of this work is on patterns and trends in the data, taking a view across countries and payment industries. This area can be thought of as a middle layer between raw data and economic analysis of causes and effects in payments. The aim is to provide a better basis for deeper analysis into underlying drivers, but also to point out gaps in the data and suggest interesting areas for further analysis.

The book consists of three parts that cover:

I. Cash and how we use it in everyday transactions
II. Other retail payment instruments: consumer choice, adoption and standards.
III. Wholesale and bank-to-bank payments: statistical distributions of payment and bank size, as well as interbank network topologies.

When looking at payment statistics across instruments and countries, some interesting overall patterns can be observed:

- Cash is (still) pervasive even in the most advanced economies. It is used for the majority of transactions, and also plays an important role in the grey economy
- While electronic instruments like cards are being adopted across the world, old habits like checks and

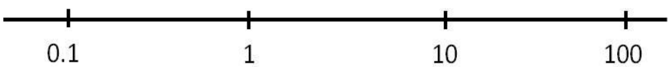
cash die hard. In fact, in spite of their rapid adoption, new instruments are far from pushing out cash, even at the point of sale

- Payments is still a national business, only a small fraction of payments are cross-border, and there are significant, and persistent, differences in the use of payment instruments across countries
- Banking and payments are characterized by heavy concentration around a few large payments and a few large and well connected banks. The data justify focus on such systemically important payments and banks.

This book uses basic tools that do not require any knowledge of mathematics. Where mathematical concepts are used, it is explained in boxes like the one below.

Logarithms

Logarithms turn exponential series into linear ones: we can write the series 0.1, 1, 10, 100, 1000 as 10^{-1}, 10^{0}, 10^{1}, 10^{2}, 10^{3}. The little exponents $^{-1}$, ^{0}, ^{1}, ^{2}, ^{3} are called the logarithms of the series. So the logarithm of 10 is 1, the logarithm of 100 is 2, etc. Since many quantities such as the size of payments cover multiple orders of magnitude, logarithms come in handy. For example we will use logarithmic scales that can cover multiple orders of magnitude in a compact way:



Note that a logarithmic scale does not have a zero: as we go left we encounter 0.01, 0.001, etc., but we never reach 0.
The above Logarithms have base 10, which we denote as *log* throughout this book. We also use natural logarithms that have base *e* which we will denote as *ln*. See the inset in chapter 6 for more on the number *e*.

PART I: CASH

This first part looks at the most basic of payment instruments: cash. While cash is by far the most common payment instrument, its usage is not systematically registered so we have to rely on survey data. These reveal the following patterns in cash usage:

- The average OECD inhabitant makes anywhere between 150 (Denmark) and 500 (Germany) cash transactions per year[1]
- The average transaction is around 10-15 euro, but the distribution is quite skewed: the modal ('typical') transaction is much lower: around 2-4 euro[2]
- The average wallet contains around 50-100 euro, typically 10-15 coins and 5-7 notes[3]
- The content of our wallets represents only 5-10% of all currency in circulation.

The remainder of this first part examines some of the questions surrounding cash:

- What are optimal note and currency denominations: do we need a $3 bill? (no we don't)

[1] Denecker and Savardy (2009) give cash transactions as percentage of total transactions for about 20 countries. These were combined with BIS data on non-cash transactions to give the quoted figures.
[2] The modal transaction size is the one that occurs most frequently. The size of payments will be further discussed in section 10. While only a few of such surveys were conducted prior to 2000, recent years have seen a surge of well conducted research into cash usage in multiple countries. For a good overview of these, see Jonker, Kosse (2012).
[3] Kippers (2004, p56) and Schneeberger and Suß (2007).

- Only about 5-10% of the currency in circulation is in people's wallets. Where is the rest? (abroad and in the grey economy)
- What is the velocity of money: how often does a dollar bill change hands? (about once a week)
- Where do people spend cash: close to home, further away, abroad? (mostly close to home, with the occasional long distance trip).

1 The size of cash payments: small is beautiful

How big is the average cash payment? Surprisingly small: the modal transaction, the most common size, is less than 5 Euro or Dollar. Unlike most other instruments, cash transactions are not recorded so we have to rely on surveys for the amount and size of cash transactions. Fortunately, surveys using different approaches in various (OECD) countries yield comparable results.

Figure 1 shows the distribution of transactions sizes from one of these surveys, representing a total of 2047 cash payments made by Dutch consumers.

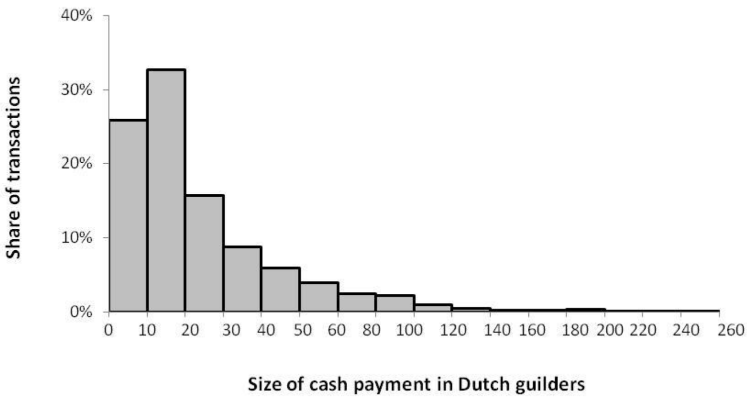


Figure 1: Frequency distribution of cash transactions[4]

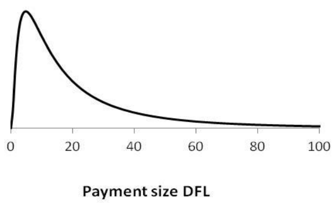
Transaction size follows something of a bell curve, but the distribution is tilted to the left: the average transaction in the sample is around DFL 25 but the median transaction is only DFL 15 (by definition half of transactions are bigger than the

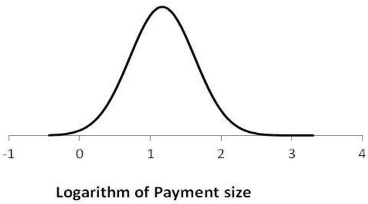
[4] From Boeschoten and Fase (1989). A Dutch guilder is about half a euro

median, the other half are smaller). On the other hand, there is a ‘long tail’ on the right: 2.5% of transactions are bigger than DFL 100 and 0.3% are over DFL 200. In fact transaction size follows what is called a Log-normal distribution (see inset), which has a “bump” near zero and then a long tail on the right.

The Log-normal distribution

Several researchers have found that the size of cash payments follow a Log-normal distribution. A variable *x* is said to follow a Log-normal distribution if its natural logarithm, denoted *ln*(x), follows a Normal distribution, also known as the bell curve. It has the same parameters as the underlying Normal distribution: μ for place and σ for width.[5] The below figure shows the fitted curve for payment size (in Dutch guilders), with $\mu=2.7$ for place and $\sigma=0.9$. On the left is the “normal” histogram of this distribution. On the right is what you get if you were to plot a histogram of the logarithm (base 10) of payment size.





The Log-normal distribution has been observed in anything from the size of living things (including the weight and length of humans) to farm sizes and the number of words in sentences written by G.B. Shaw.[6]

It turns out that transaction sizes of non-cash instruments also follow a Log-normal distribution. The size of a SWIFT or Fedwire transaction is more than 10,000 times bigger than a cash transaction, but they still follow that same Log-normal distribution, as we will see in chapter 10.

The Netherlands has a tradition of research into cash payments, which allows us to compare cash patterns over time.

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[5] For a good introduction, see Aitchison and Brown (1957).

[6] Limpert, Stahel (2001)

Table 1 summarizes the results of three such cash surveys covering the period 1987-2010.

Table 1: Comparison of Dutch Cash usage surveys[7]

Year	Cash payments per person per year	Average value[8]	Median Value	$\hat{\mu}$[9]	$\hat{\sigma}$
1987	574	19	11	2.4	0.9
1998	n/a	10	6	1.8	1.0
2010	316	12	6	1.8	1.0

A few interesting observations can be made from this. First, the overall number of cash transactions has gone down significantly. This pattern is observed across OECD countries and is of course due to the advance of electronic instruments, notable the debit card (see also chapter 6). The average transaction amount also declined, as one would expect since electronic instruments tend to be used for larger payments. But notice how the overall variance ('spread of the sample') *in*creases. This could indicate that most substitution by new instruments takes place in the *middle* range (say 30-100 EUR) while cash continues to be used for very small but also for very large transactions, for example in the grey economy (see also chapter 3).

[7] The three surveys are described in Boeschoten and Fase (1989), Kippers, Van Nierop (2003) and Jonker, Kosse (2012).
[8] The values for mean and median have been converted to 2010 Euro.
[9] These estimates have been obtained by the Maximum Likelihood method.

A comparison of the histograms of the 1987 and 2010 surveys confirms this (Figure 2). In relative terms, the extremes have been stable with relatively large reductions in the middle.

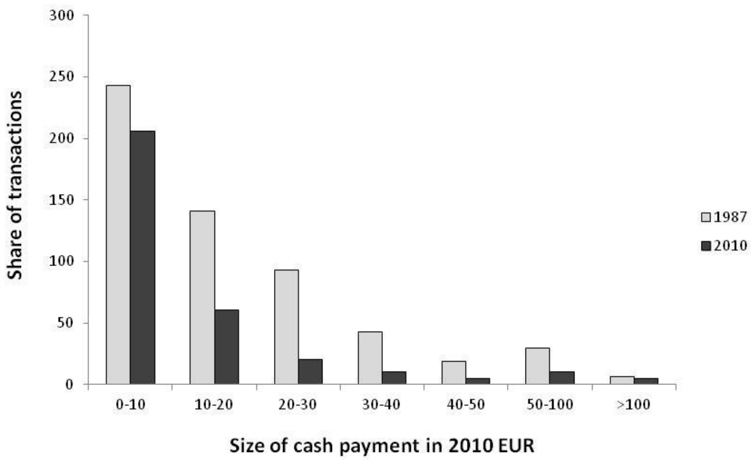

Figure 2: Comparison of Dutch 1987 and 2010 cash payment sizes[10]

[10] The figure is based on the bins used in the 2010 study of Hernandez and Jonker (2011). The bins of the 1987 study (Boeschoten and Fase 1989) were in guilders, so these have been converted to euro and corrected for inflation. The number of payments in each converted 1987 bin have then been reassigned to the 2010 bins by interpolation.

2 As phony as a $3 bill? Coin and note denominations

If you are a central bank, how do you design an optimal set of coins and notes? In particular, what denominations should you select? As we saw in the previous chapter, cash payments span multiple orders of magnitude: from 0.01 cent to hundreds of dollars. The denominations should allow for paying all such amounts in a convenient way.

This problem has, of course, been solved numerous times in practice. Table 2 shows the Roman coin system that was largely based on powers of 2.

Table 2: Roman coins denominations, Augustan values

Name	Value in Sertertius	Value in Quadrans
Aureus	100	1600
Quinarius Aureus	50	800
Cistophorus	12	192
Antonianus	8	128
Denarius	4	64
Quinarius Argenteus	2	32
Sertertius	1	16
Dupondis	½	8
As	¼	4
Semis	1/8	2
Quadrans	1/16	1

At the time of Charlemagne, the penny was the dominant minted coin and contained about 1.7g of silver. Large quantities of pennies were counted in dozens (the shilling) and score dozens (the pound).[11] A British pound sterling being 240 pennies thus was indeed a pound of silver. This system was still used by the UK, prior to going decimal in 1971. It has been argued that this system was close to powers of 3 (Table 3).[12]

[11] Sargent and Velde (1997), p17.

[12] Telser (1995) converts all denomination to pence, so that 1 shilling = 12 pence; ½ crown = 2½ shilling = 30 pence, etc. He omits the farthing (¼ pence), half-penny and two-pence.

Table 3: Denominations of UK coins and notes (in pennies)

Powers of 3:	1	3	9		27	81	243
UK prior to '71:	1	3	6	12	30	60	240

Most of today's systems are decimal in the sense that they have denominations for the powers of 10 ($^1/_{100}$, $^1/_{10}$, 1, 10 etc.). To fill the gap between these powers of ten, most currencies use some form of the so called *binary-decimal triplet* {1, 2, 5}. Some 20 currencies use this system in its pure form, including the euro which has 5 of these triplets going from a 1 Eurocent coin all the way to a 500 euro note. Less common, but still frequently used, are the *fractional-decimal triplet* {1, 2½, 5} and decimal pairs like {1, 5}, {1, 2½}. Many currencies use a mixture of these. The US dollar, for example, has a 25¢ coin, a $2 and $20 note but neither a 50¢ coin nor a $50 bill. And there is, of course, no $3 bill. In fact, only very few currency systems have coins or bills that are powers or multiples of 3.[13]

So we know it works in practice, but does it work in theory? Are these systems indeed optimal? This problem turns out to be harder than it looks. It has inspired significant modelling effort and some fierce academic debate between two different schools of thought.

The first school looks for the minimal set of different denominations that can make any payment, assuming each denomination (like weights) can be used only once in a payment. It turns out the optimal denominations are the powers of 3, hence the old UK system came close.

A second school of thought looks at the *total* number of tokens (coins and notes) needed for a transaction, i.e. allowing for the use of multiple coins of the same denomination. If exact payment is required then it can be shown that the system with

[13] A 3 lek note in Albania, a 3 peso note in Cuba, a 3 bani coin in Rumania, a 3 rouble note in Russia and 3 Bahamian dollar note. These are the exceptions, not the rule. Wynne (1997).

base 2 is the most efficient, i.e. yields the lowest number of tokens across all transaction sizes.[14] Under this view, the Romans got it right.

Blâchet's weight problem

Finding the minimal set of different denominations that can make any payment (allowing for change) is mathematically related to the weight problem of Bâchet: break a 40 kg stone into as few pieces as possible so that you can weigh any whole-kg amount between 1 and 40 kg using only a two-scale balance.[15] The answer is to use powers of 3 and break the stone into 4 pieces weighing 1, 3, 9 and 27 kg.

The analogy to payments is as follows: the object to be weighted corresponds to a transaction price which has to be paid in cash. The weights correspond to the coins and notes used for payment, and the weights added to opposite pan (the pan holding the object to be weighted) correspond to change given in the transaction.

By this analogy, the best denominations for coins and notes would be powers of three: 1, 3, 9, 27, 81, etc.

Things are more complicated if change can be given. Quite some effort has been put in computer simulations to find the system that would result in the lowest number of tokens (coins or notes) across a range of transaction sizes. It turns out that the Roman system (powers of 2) is more efficient than the old English system (powers of 3).[16] But the best system would be based on powers of 1.53, which gives something like {1, 2, 3, 5, 8, 12, 19, 30, 46, etc.}.[17] If this seems arithmetically challenging, there is hope: there is another theoretical optimum for a system that uses powers of 2.16. Using this value yields denominations that are very close to our decimal systems (see inset).

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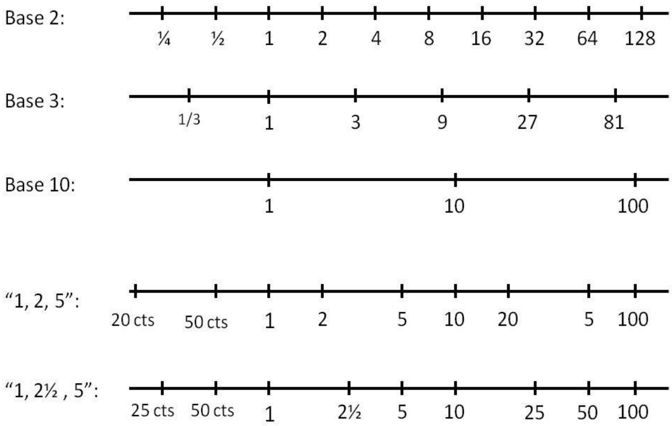
[14] Caianiello, Scarpetta et al. (1982).
[15] Telser (1995).
[16] Van Hove and Hendels (1996).
[17] Bouni and Houy (2007). Denominations are rounded down to the next integer.

Even spacing on logarithmic scales

As was argued in the introduction, logarithms offer a convenient way to deal with variables, such as payment size, that span multiple orders of magnitude. Note how most currency systems are evenly spaced on logarithmic scales:



A systems that would evenly space 2 extra denominations between the powers of 10 would use powers of $\sqrt[3]{10} \approx 2.154$; both the {1, 2, 5} and the {1, 2½, 5} systems get close to this.

There is of course something to be said for ease of arithmetic, which may have played a big role in favour of systems that include multiples of 10. It seems amazing that both the Romans and the English ruled the world with denominations that most of us would find arithmetically challenging.

One thing seems conspicuously absent in most of the research: the actual distribution of payment transaction sizes. The research covered in this chapter assumes transaction sizes are uniformly distributed. In fact, transaction sizes follow a Log-normal distribution which is heavily skewed towards smaller sizes: the modal (most frequent) payment size is close to $2. One could ask, therefore, why the US has no coins for 2 cents and 50 cents, while it does have $20 and $50 notes.

3 What happened to all the $100 bills? The mysterious case of the missing cash

We know from consumer surveys that the average consumer holds about 50-100 in cash in their wallet. And yet there is some $3000 in circulation for every American and €2,450 for every euro zone inhabitant.[18] Japan and Switzerland have even larger amounts per person. Even if we account for cash held by businesses (about half of what consumers are holding) there is a huge gap, with most of the missing currency in the higher denominations: 75% of US currency by value is in $100 notes and a third of Euro currency is in €500 notes. Where is all this cash?

This question has generated significant research; currency is an important part of monetary policy, so Central Banks would like to understand how and where it is being used in economic activity. The outstanding currency also generates significant seignorage income: the notes and coins in circulation are in effect an interest-free loan from the public to the currency issuer. The $800 billion in circulation save the US government some $20 billion per year in interest. If the demand for currency were to suddenly decline, the treasury would have to borrow this money somewhere else and pay interest.

Two places have been suggested for this missing cash: abroad and in the underground economy. There is good reason to suspect that a significant amount of at least some currencies is held abroad. Several Latin American countries, notably Argentina, use US dollars de facto as a domestic currency. In several Eastern European countries, notably former Yugoslavia, the Deutschemark and later the euro were similarly used domestically.

[18] According to BIS, in 2010 there were 2400 Euro in notes and coins per capita in circulation, meaning less than 5% of this was in consumer wallets.

Hard data on this foreign usage are very scarce. For US dollars we can use data on shipments of physical currency to and from foreign countries as an indicator; any shipment of over $5000 has to be reported to US authorities. Analysis of these data suggests 30-37% of dollars are held abroad, with the main countries being Russia, China and Argentina. By these calculations each Argentinean would hold $1000.[19]

For Deutschemark no such shipment data are available but the unification of Germany provided an interesting clue: 10 billion DM were supplied following unification, or about DM 650 per inhabitant of former East-Germany, much lower than the amount in circulation for each West-German at the time. Correcting for standard of living, this suggests that some 30-40% of Deutschemarks were held abroad.[20]

Even if a third of currency is abroad that still leaves a significant amount of cash unaccounted for. The general assumption is that hoarding plays a role, but much of it is used in drug-trade and unreported economic activity. An IRS report on non- or mis-reported tax income gain, puts income from the grey economy at 17% of total US GDP in 1988. We can extrapolate from there if we assume that the ratio of cash for official use to bank balances is constant, and that any extra growth in currency must be due to growth of the informal sector of the economy.[21] The ratio of currency to bank deposits grew from 31% in 1988 to 38% in 2008. Assuming the excess growth comes from illicit usage, the underground economy grew from 17% of GDP in 1988 to about 22% in 2008.

So where does this leave us on the mystery of the missing currency? With 15% of currency used for cash transactions, some 20-25% used in the informal economy and 30-40% held

[19] Feige (2009).
[20] Seitz (1997).
[21] This approach is proposed by Feige (2009) and the figures are taken from his paper.

abroad, we are still short some 15-35%, presumably stuffed in mattresses and various lockboxes.

One consequence of the fact that relatively little currency is used for actual legal cash transactions is that governments need not be overly worried that their seignorage revenues will disappear due to the rise of electronic payment instruments such as debit cards. At the same time the impact on outstanding currency stocks appears to be almost unobservable. This seems logical since most of the outstanding currency is in larger denominations which are not commonly used for transactions.

In fact, several researchers have looked into fluctuations in currency outstanding by size of denomination. They distinguish the amounts of large, medium and small denominations.[22] Analysis shows that the amount of large denominations is negatively correlated with the interest rate: the higher the interest rate, the lower the amount of large notes in circulation. The amount of small notes and coins follows a quite different dynamic, and is negatively correlated with the use of electronic payment instruments such as debt cards.

Hence large and small denominations fulfil two very different functions of cash. Small denominations are a medium of exchange while large denominations are a store of value.

[22] Amromin and Chakravorti (2007) and Fisher, Köhler et al. (2004). They define medium as the denominations given out by ATMs, large is anything above that and low is anything smaller than that.

4 How fast is that buck? The velocity of money

How often does a dollar bill change hands? This quantity, known as the velocity of money, is relevant because it influences inflation: higher velocity of money has the same effect as an increase in the quantity of money; if money works harder, you need less of it (see inset).

The relativity of money

Economics has its own version of Einstein's $E=MC^2$. It is Irving Fisher's equation of exchange:

$$MV = PT$$

Here *M* denotes the total money supply, *V* the velocity of money, *P* the price level and *T* the amount of transactions. The formula is simple and intuitively appealing, but estimating the actual values for these variables is not straightforward, to put it mildly. Perhaps the most enigmatic of all is V, the velocity of money: how often does a dollar or euro change hands? We can rewrite Fisher's equation to $V = \frac{PT}{M}$, but while *M* is measurable, PT is much harder to obtain. We measure *changes* to *P* through price level indices, but for the formula we need the value of all transactions, i.e. the average price times the volume of all transactions, including intermediate goods and asset transactions.

One way to estimate the speed of cash is to look directly at consumer cash behaviour. A Federal Reserve survey, for example, that finds that physical currency turns over 55 times per year, i.e. about once a week.[23] We can combine this with data on banknote fitness and replacement by the Federal Reserve. The Federal Reserve inspects notes returned by banks and replaces the ones that are worn out. It turns out that that lower denomination notes have a relatively short lifetime of about 1.5 years, while a $100 bill last 7.5 years.[24] Assume that each note is used for the same amount of payments before it is

[23] See Avery (1986).
[24] Analysis and figures from Feige (1989)

worn out. This gives an average turnover of about 110 times/year for \$1 and \$5 notes; this would imply that each such note is used for a payment about twice a week. For \$20 notes this is 75 times/yr or once every 5 days, while the \$100 notes are used much less: 20 payments a year, or once every 2.5 weeks.

It is interesting to compare the velocity of cash with the velocity of bank deposits. In 2010, US bank deposits stood at \$ 7.6 trillion.[25] The total volume of Chips and Fedwire transfers for that year was \$965 trillion; if we add ACH and check clearing volumes we get around \$1000 trillion. This gives a velocity of 1000/8.4 = 138 times/year, more than double the velocity of cash. This means that bank deposits “work at least twice as hard” as cash.

It is equally interesting to estimate the velocity of the “missing cash” used in the underground economy (as discussed in chapter 3). Cash held by consumers and businesses accounts for some 15% of total currency, with another 30-37% residing abroad. This implies that about half of all currency would be used in the underground economy, about 3½ times the cash used for official purposes. If the underground economy really is 22% of the official economy, then the underground cash is not very fast: its velocity is $\frac{22\%}{3.5} \approx 6\%$ of the official speed. This corresponds to 3.3 payments per year, or less than one transaction per quarter. Even if we assume all underground transactions are made with relatively slow moving \$ 100 notes, these notes have a relaxed life compared to their official cousins who are used in transactions every 2.5 weeks, 6 times as often. Presumably these unofficial \$100 notes spend most of their life in vaults, storing value and avoiding taxes and drug enforcement officials.

[25] US *M2* was 8\$.4 trillion. This definition of money includes both currency in circulation and bank deposits. Currency (coins and notes) stood at \$800 billion leaving \$7.6 trillion for bank deposits.

5 Where's George? The wanderings of individual notes and coins

Where do we spend our money? Close to home, further away, abroad? While little is known about the geography of spending, we have a few indirect sources of data. For example we know that less than 5% of all non-cash transactions are cross-border. We also have indications that when travelling abroad, consumers tend to use cash more frequently than at home.[26] And we have two intriguing sources about travel behaviour of cash.

The first one is the introduction of the physical euro. Euro coins bear an emblem that differs by country (e.g. the profile of the king for Belgian coins and a harp for Irish coins). With the introduction of the physical euro on Jan 1st 2002, each euro country was supplied with a set of Euro coins bearing the emblem of that country. We would expect a mixing of coins over time, with 'foreign' Euro coins become ever more prevalent in wallets.

Significant effort has been put in modelling this process and in collecting actual data by asking volunteers to regularly check their wallet and count the coins by origin.[27] In general these data are unreliable and not granular enough to allow for the fitting of the sophisticated models. However some interesting general facts can be observed.

In the first place, distance seems to play a role. This is perhaps best illustrated by the map in Figure 3, which depicts the share of Austrian euro coins in Germany in February 2002, two months after their introduction. The Austrian euro coins are

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[26] See Jonker and Kosse (2008). They find that limited cross-border acceptance of debit cards hampers its cross-borderusage.

[27] Such models are applied to Euro coin mixing by Stoyan, Stoyan et al. (2004), Blokland, Booth et al. (2002) and Seitz, Stoyan et al. (2009).

spreading over Germany from the South, presumably brought home by Germans returning from ski-trips.

Secondly, diffusion differs by denomination. Several sources confirm that €1 and €2 spread about twice as fast as 5 and 10 cent coins, and almost three times as fast as 1 and 2 cent coins.[28] Why this is the case is anyone's guess. Perhaps because people are more aware of the larger coins, hence tourists are more likely to take them home.

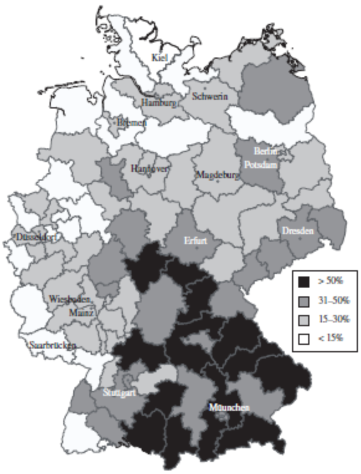


Figure 3: Spatial distribution of Austrian coins in Germany as of January and February 2002[29]

Finally, the mixing was initially fast but then slowed down. For example, based on early observations it was predicted that over half of the coins in Germany would be foreign after 6 years. In

[28] Schneeberger and Süß (2007), Blokland, Booth et al. (2002) both report this phenomenon.

[29] Data from Stoyan, Stoyan et al (2004)

fact, a recent study found that in 2008 still 75% of all €1 coins in German wallets were of German origin.[30]

A second, equally intriguing, source of data is a project called "Where's George". This project tracks individual US dollar bills. Volunteers enter the serial number of the bills in their wallet using a website. This allows for the tracking of some 450,000 individual bank notes. Analysis of this data shows that over half of the notes travel less than 10 km between to reports (typically a few weeks) but there is a "long tail" of notes that travel much longer distances: 800km or more.[31]

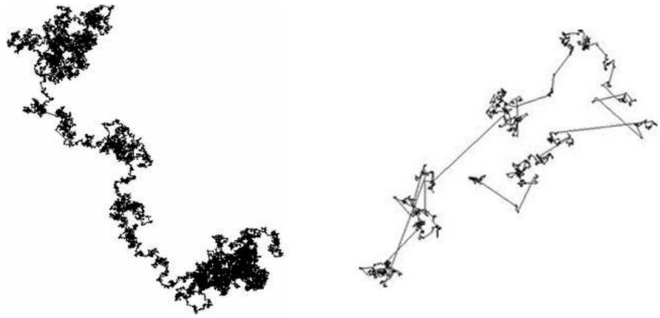


Figure 4: Random walk (left) versus Lévy flight (right)

These "long tails" are indicative of a dispersion process that differs from the traditional random walk where distance and direction travelled during each period follow a Normal (bell curve) distribution. The left side of Figure 4 shows an example of such a traditional walk, also known as 'Brownian motion'. Bank notes follow a pattern like the one on the right of Figure 4: very local movements, interrupted by long distance travel. These patterns are called Lévy flights.

[30] For example compare Stoyan, Stoyan et al (2004) with Seitz, Stoyan et al. (2009).

[31] Brockmann (2006). This long tail follows a Power-law. Power laws are described in Part III of this book.

Obviously, notes and coins do not travel by themselves, but are typically carried by humans. As such the whereabouts of George provide give a good source of information on human travel patterns.

Part II: NON-CASH PAYMENT INSTRUMENTS

Cash is, of course, not the only way to pay. There are many alternative instruments such as checks, credit/debit cards, ACH transfers, etc. We have much more reliable data on these instruments than on cash. We know, for example, the number of transactions and their value. Analysis of this data reveals some interesting patterns:

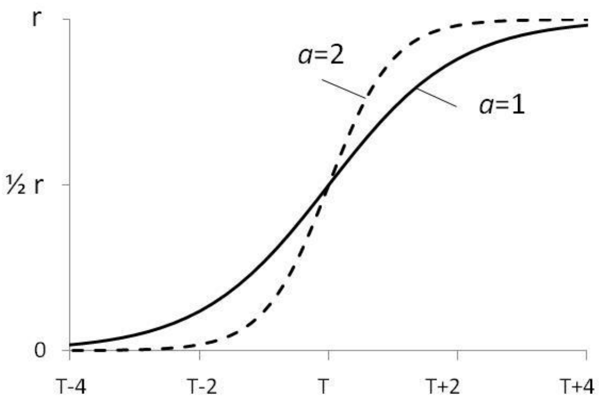
- electronic payment instruments, notably debit cards, have been enthusiastically adopted across both developed and developing economies
- but old habits, notable the use of cash and checks die hard
- there are big and persisting differences between countries in the use of payment instruments, with little convergence
- even in the most advance economies, cash still dominates spending patterns in terms of number of transactions
- cash will continue to play a big role in payments for most of the 21st century unless there is a revolutionary increase in the adoption of electronic payments

Adoption curves

Technology adoption of technologies tends to follow an S-curve. This is typically modelled with a logistical curve: at any time *t* the number of users is equal to:

$$\frac{r}{1+e^{-a(t-T)}}$$

The crucial parameter here is *a* which determines the shape and steepness of the curve. For *a>0* we get adoption, for *a<0* we get a downward sloping S-curve that can be used to model dis-adoption. The higher variable *a*, the faster the adoption. The figure below shows two S-curves, one with *a*=1 and a steeper one with *a*=2. Initially adoption growths exponentially a rate *a*. The growth rate then slows down until the number of users asymptotically reaches the ceiling *r*. Parameter *T* shifts the curve from left to right, and denotes the year/period in which adoption reaches 50% of the ceiling *r*. Finally *e* is Euler's constant (see inset in next chapter).



6 Cash or Card? The growth of electronic instruments

The use of payment instruments has undeniable changed over the past 20 years. Figure 5 shows the development of transactions per person in the Netherlands. In 1988, consumers had a choice between checks and cash at the point of sale, with cash used for the vast majority of these transactions. 22 years later, checks have disappeared and cards are now used for a significant portion of purchases; the vast majority of these are POS debit transactions, whose growth has been spectacular. But even with that growth, cash is still used for two-thirds of purchases and for half of all transactions[32]. For non-POS payments, transfers have continued to grow and direct debits have established themselves as a solid alternative.

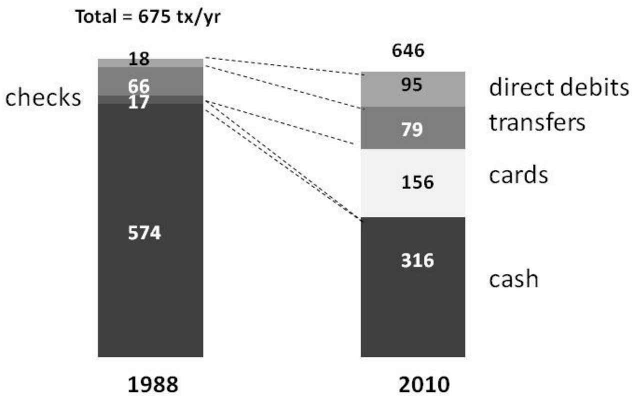


Figure 5: Use of non-cash payment instruments in the Netherlands, transactions per capita

The spectacular growth of POS debit is not unique to the Netherlands but can be observed across the developed world, and even in developing countries.

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[32] Since cash transactions tend to be relatively small, their share in the value is lower but still almost 40% of all POS transactions by value.

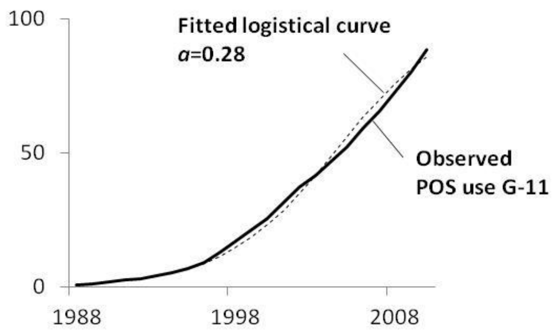


Figure 6: debit card transactions per person

Figure 6 shows the growth of transactions per capita the G-10 countries, where per capita usage has gone from less than 1 in 1988 to almost 100 in 2010. The figure also shows a fitted 'S-curve'.[33] The fitted curve has steepness parameter a=0.28, a midpoint around T=2004 and a ceiling of r=105 transactions per person.

In line with the S-curve model, annual growth in G-10 debit card adoption slowed down as adoption increased. It went from 32% in the early years (1988-1995) to 22% in the following 7 years to 2002, then further decreased to 10% in the 8 years to 2010. The question remains however: how much further will it grow? The fitted G-10 adoption curve has a ceiling of 105 transactions per person, but this seems questionable since in several countries debit usage is already well above this ceiling. The US for example, had 160 transactions per person in 2010. Also, the G-10 adoption of POS debit is slowing down but shows few signs of tapering off.

POS debit replaces cash (and in some countries checks), and while cash transactions have been decreasing there is quite some way to go. In the Netherlands for example, per capita

[33] Data from BIS. The G-10 countries are Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, UK and US (11 in all; Switzerland was added later but the name G-10 remained).

cash transactions stood at an estimated 450 in 2010, more than three times the number of debit card transactions. On the other hand, unless the pace of POS-debit adoption increases dramatically, it will take another 20 years before it surpasses cash as the dominant medium at the Point of Sale, and much longer before it makes cash disappear.

Figure 7 shows growth of POS debit in BRIC countries where usage is still much lower with 12 transactions per person in 2010, about equal to where the G-10 was in 1997.[34]

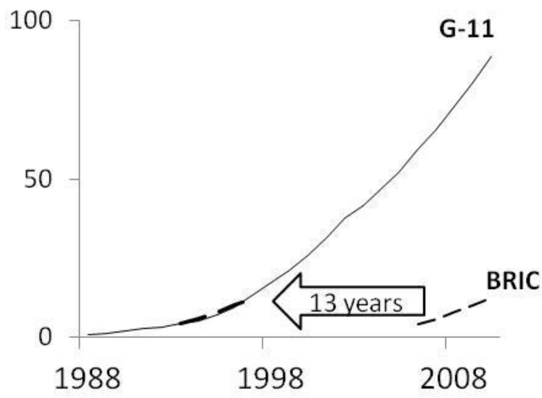


Figure 7: POS debit transactions per person in BRIC countries

Fitting a curve to growth in BRIC countries yields a steepness of *a*=0.29 and a midpoint at *T*=2017. The steepness parameter is remarkably similar to the curve for the G-10, with initial growth only slightly higher. The main difference is the shift parameter *T* which suggests the BRIC countries will reach the mid-point to saturation in 2017 versus 2004 for the G-10. Growth in the BRIC countries appears to follow almost exactly the same path as it did in the G-10 countries some 13 years earlier.

[34] BRIC countries are Brazil, Russia, India, China. Data from BIS.

_e_ and interest compounding

If the annual rate on your deposit is 12%, but you get 3% each quarter, the effective rate is higher due to compounding: at the end of the year you have $(1.03)^4$ ≈\$1.1255: the effective rate is more than half a percentage point higher than the nominal rate of 12%. More generally if an annual nominal interest rate *i* is paid *n* times a year, the effective rate is: $i^{eff} = \left(1 + \frac{i}{n}\right)^n - 1$.

As we compound more often, the effective rate becomes higher. For monthly interest, our 12% would become $(1.01)^{12}$ -1≈ 12.683%. What if interest would be compounded continuously? Is there a limit if *n* goes to infinity?

For *i*=100% we know the answer due to the definition of Euler's number *e*:

$$\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e \approx 2.71$$

So 100%, continuously compounded, gives an effective rate of 171%. From the definition of *e* we can deduct the limit for other rates *i* as well: $i^{eff} = e^i - 1$.

So continuously compounding 12% per year gives an effective rate of $e^i - 1 \approx 12.750\%$.

7 Rumors of the death of checks are greatly exaggerated.

Just as universal as the rise of POS debit has been the decline of checks. Figure 8 shows the development of checks written per person in the high check usage countries, as well as three low usage countries. Check usage in the four high usage countries appears to be in constant decline: about 5% per year in Canada and France, around 8% per year in the UK and the US. In the UK the rate of decline seems to be accelerating, although the instrument maybe around for a while. In 2011, the UK banks announced their intention to withdraw the check altogether only to find fierce consumer and political resistance. They were effectively forced to scrap their plans. This illustrates how difficult it is to completely retire payment instruments.

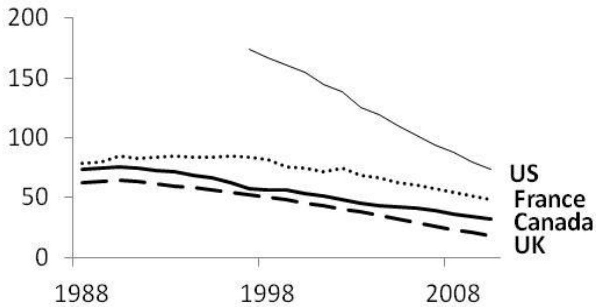


Figure 8: Checks written per person per year in high usage countries

For Belgium and the Netherlands we observe something close to a ‘reversed S-curve’, i.e. a logistical curve with a<0 (Figure 9). For Belgium the estimated values is a= -0.24, so the speed of check dis-adoption curve is comparable to debit card adoption. For the Netherlands we get a much steeper a= -0.43, perhaps because that country actively phased out the instrument altogether in 2001, in contrast to Germany and Belgium where the checks continue to be used at low levels.

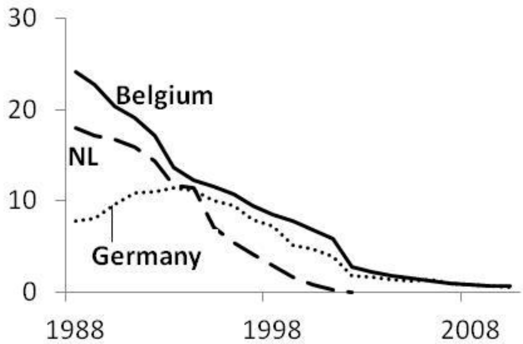


Figure 9: Checks written per person per year in low usage countries

An interesting instrument that showed S-curves both on the way up and on the way down is the Telex, pictured below.

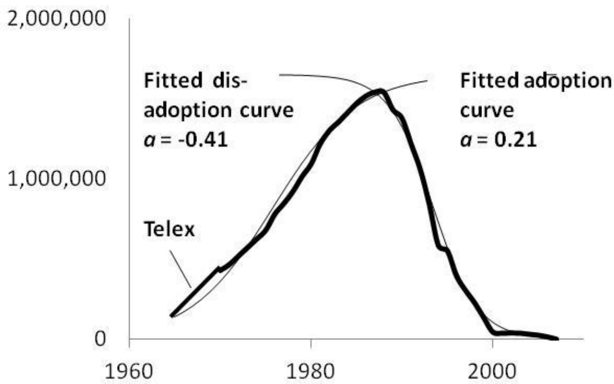


Figure 10: Number of telex lines worldwide[35]

As the telex gained adoption in the 1960s, the number of telex lines grew by over 20% each year from 1965 to 1970. From 1970 to 1980 growth continued at a slower 10% per year, slowing to 6% in the early 80s. With the rise of electronic networks like SITA (airline industry) and SWIFT (banks), the number of subscribers started to decline as of 1987. Slowly at

[35] Source: UPU

first, by 7% per year from ’87-‘92, but then faster, reaching a decline of 39% per year in the years ’02 to ‘07.

The number of subscribers appears to follow 2 S-curves: one for the original adoption and a second one for the dis-adoption as the telex was replaced with newer technologies like electronic computer networks. Figure 10 shows these two curves. The first one has a steepness parameter of a=0.21 comparable to the adoption of debit card transactions. The second one has a steepness parameter of a=-0.41, much stepper and comparable to the steepness of check decline in the Netherlands. One can think of the telex and checks as intermediate technologies: better than what they replaced (phone/mail/cash) but eventually driven out by more efficient electronic networks.

It is interesting to compare the dis-adoption of telex with the adoption of SWIFT, one of the electronic networks replacing the telex. Figure 11 shows the number of banks using SWIFT with a fitted logistical curve. The steepness parameter a has a value of 0.17, which means the adoption of SWIFT was much slower than the decline of the telex.

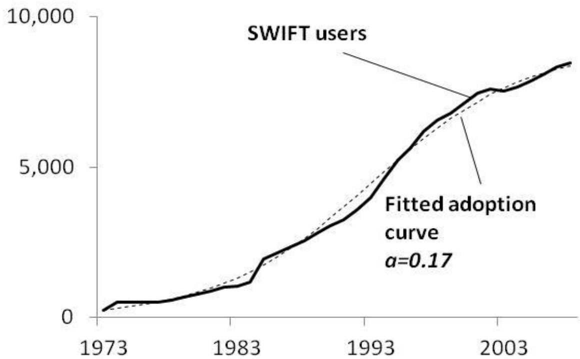


Figure 11: Number of SWIFT users

A possible explanation for this disparity could be that many SWIFT banks kept their telex lines to communicate with counterparties that were not yet on SWIFT. Once the new networks gained critical mass, there was a backlog of banks ready to decommission the telex, leading to a rapid decline.

8 Sprechen Sie cash? National differences in payment instrument usage

While all countries are adopting electronic instruments, there are significant differences in the use of payment instruments. Some of these reflect different development stages, like the adoption of debit cards in the BRIC countries (see Figure 6 in chapter 6). We see a similar pattern in the usage of cash. Figure 12 shows the usage of cash by country. BRIC countries like Brazil and Russia are still using cash for more than 90% of all transactions. Perhaps more intriguing are Japan, Italy, Germany and Switzerland. These are well developed economies that continue to rely heavily on cash. Japan and Italy use cash for more than 85% of all transactions, Germany and Switzerland for 70-75%. Most of the other developed countries use cash for only 55-60% of their transactions.

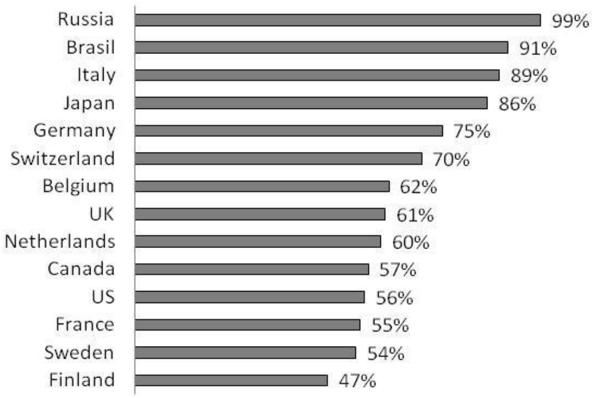


Figure 12: share of cash in total transactions, 2008[36]

There are also significant differences among countries in the types of non-cash payment instruments that people use. One of these already was apparent from the discussion on the decline

[36] Source: Denecker, Savardy (2007)

of checks: the US, UK, Canada and France continue to use checks for a significant amount of payments.

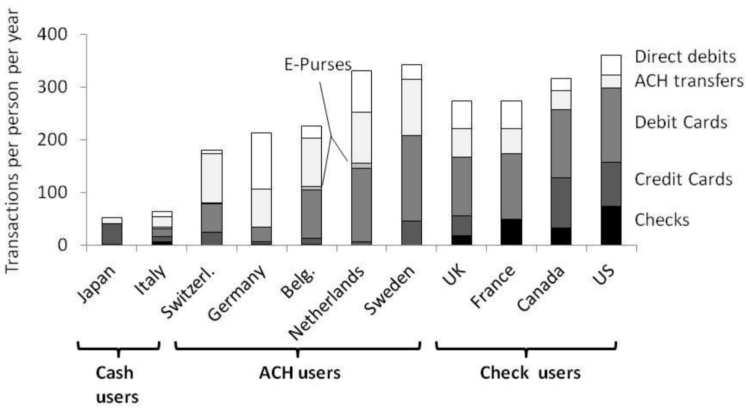


Figure 13: Use of non-cash instruments for G-10, 2010[37]

Figure 13 shows the use of non-cash payment instruments across the G-10 countries. It shows three broad categories: cash-countries like Italy and Japan with low usage of non-cash instruments, the four check countries mentioned earlier, and "ACH-countries" that rely on transfer payments and direct debits. This last category includes many Central and Northern European countries.

An interesting question is whether these differences are likely to disappear as countries move off cash and checks and adopt debit card payments. Perhaps, but it will take a long time. Figure 14 shows the development for checks and debit card transactions since 1998 for the three types of countries. In terms of POS debit card transactions, the gap between the cash countries and the others actually increased dramatically.

[37] Source: BIS Red books

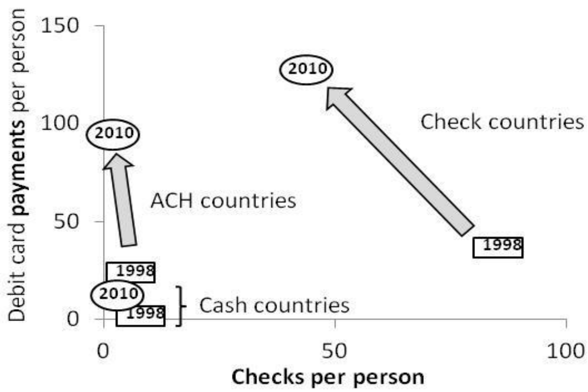


Figure 14: Transactions per person in 1998 and 2010

To some extent, this may be caused by the mechanics of the adoption process as described in the previous chapter. If two countries are on the same S-curve but one country has a head start of a few years, then the difference in usage between the two countries will initially grow; the gap will only start to narrow once the first country is well beyond the midpoint to full adoption. So it is unlikely that cash countries like the BRICs and Italy and Japan will catch up any time soon.

In terms of check use there is some convergence, but at a very slow pace. Hence we should expect countries to retain their distinct usage patterns for the foreseeable future.

Much research has been done into the causes of these differences. For example it has been argued that high cash usage may be related to high taxes and low crime rates. Several comprehensive studies have found at best a weak relationship between safety, taxation and cash usage. Differences in infrastructure, such as the availability of ATMs and POS terminals have also been offered as an explanation, but this seems somewhat circular: more ATMs may well mean more cash usage, but a high usage of cash means that a larger number of ATMs make economic sense. Finally, the pricing of instruments could explain the differences.[38] But most payment instruments, including cash, are offered for free to consumers with most of the costs put on merchants or banks. In addition much pricing is hidden and implicit, such as ATM 'roaming' fees and hefty bounced check charges in the US.

An better explanation could be that payment mechanisms are subject to network effects: the more people are using a certain instrument, the more valuable the instrument becomes to all users. We know that such networks are subject to lock-in: once users have settled on a standard it becomes difficult to switch to another one, even if it is better.

Network effects can explain some of the patterns observed in the previous chapters:

- The S-curve type adoption pattern: early adopters establish some critical mass; after that the product becomes attractive to the masses and larger groups of users join.
- Dis-adoption is hard and existing instruments tend to continue to be used for a long time after better alternatives are available: existing instruments have the benefit of critical mass which the newer instruments still need to establish.

Usage patterns differ by country and these differences are persistent: once an instrument has critical mass in a country, why would it change to a standard from another country, especially since cross-border transactions are such a low fraction of total?

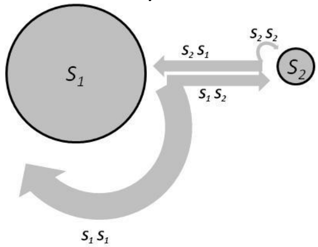
[38] For example Humphrey, Pulley and Vesala (1996).

9 The payment world isn't flat

How does money flow between countries? If the world were flat (in the sense of T. Friedman's book) transactions patterns would be global. In such a world the majority of transactions would be cross-border. In fact, the share of domestic transactions (as a percentage of all transactions) would be equal to the sum of the squared population shares of all countries (see inset).

If transaction patterns were global…

If transactions were truly global, a Londoner would be as likely to pay a South African as someone in his own city. The below figure shows such a pattern for a world with a big and a small country with population shares s_1 and s_2. The inhabitants of country 1 will initiate a share s_1 of all transactions. The majority, namely s_1^2, are domestic and the remaining $s_1 s_2$ are with country 2.



More generally, for N countries with population share $s_1, s_2,..,s_N$, it follows that $f_{ij} = s_i s_j$ where f_{ij} denotes the flows between counties i and j, measured as a proportion of total global flows. The global share of domestic transactions is then equal to $\boldsymbol{\Sigma} s_i^2$ and the share of cross-border equal to $1-\boldsymbol{\Sigma} s_i^2$.

For the world population, this sum of squared shares is about 8%.[39] So if global payment patterns were random, one would

[39] This concentration index is largely driven by China and India who account for 0.07 of the 0.08. We could also take the shares of GDP instead of population, in which case we get 0.07 instead of 0.08. Smaller, but not materially different.

expect that about 1-8%=92% of all payments in the world to be cross-border with the remaining 8% being domestic.

Reality is not even close to this prediction. In fact, it is almost the reverse: at most 5% of all payments in the world are cross-border, the rest are domestic.[40] Clearly transactions patterns are heavily domestically biased.

While transaction patterns are local, we do find that country size plays a role in cross-border transactions. A relationship called the payment gravity law (see inset) has been used to model equity flows between two countries.[41] This gravity law has also been found in large value payments flows on the TARGET2 system and international flows of US currency.[42] [43]

Payment gravity

Financial flows between countries, for example cross-border equity flows, have been modelled using the following relationship:

$$f_{ij} = a\frac{m_i m_j}{r_{ij}}$$

Here f_{ij}, represents the flows between countries i and j, m_i and m_j are some measure of their size, r_{ij} is their distance and a is a constant. This model is known as the payment gravity model, due to its similarity to Newton's gravity law.[44]

Applying the gravity model to SWIFT corresponding banking flows between the 50 largest countries yields the following relationship:

[40] BCG world payment report

[41] Portes and Rey (2005). They use the stock market capitalization of each country as an approximation of their mass, and the geographical distances between the financial centers as a measure of r_{ij}.

[42] For TARGET see Rosati and Secola (2005), who measure country size by the total balance sheet of the financial institutions in each country.

[43] For US dollar bills see Hellerstein and Ryan (2009).

[44] Albeit that Newton takes the square of the distance.

$$f_{ij} = 2.32 \frac{(m_i m_j)^{1.12}}{(r_{ij})^{1.21}}$$

Here f_{ij} denotes the value of all the SWIFT payments exchanged between countries i and j in millions of US dollars per year. m_i denotes the total assets of the banking system in country i in trillions of US dollars and r_{ij} is the distance between the financial centers of countries i and j measured in '000 of km's.

The estimated parameters are quite close to the theoretical model: the exponent for $m_i m_j$ is 1.12 and the exponent of r_{ij} is 1.21 where the model specifies a value of 1 for both.[45]

[45] The relationship was estimated in log form. The parameter estimates were all significant at the 1% level with an overall R^2=0.62%.

Part III: Interbank payment systems

Part III examines the statistics of the interbank payment systems such as TARGET, Fedwire, CHAPS and SWIFT. This leads to some interesting findings:

- The size of interbank transactions follows the same statistical distribution as cash payments, albeit with different parameters.
- Interbank networks have the same topology as the world wide web, directorships of Fortune 500 companies and authors of articles in scientific journals: most banks have few links, while a few banks have a large number of links. Such networks are efficient but vulnerable to failure of a single highly connected node
- Bank size itself (measured in size of balance sheet) is similarly concentrated: most banks are small to medium, with a few very large ones.

Payment size, the number of links of banks and the size of banks all follow statistical distributions with 'fat tails' (see inset below). One consequence of this is concentration: 5% of all SWIFT payments account for 95% of the value, 20% of banks account for 80% of all counterparty links and 5% of banks account for 95% of all assets.

Fat tails; if it is bad, it is probably worse than you think

Like the Log-normal distribution, Power-law distributions have ‘long’ or ‘fat’ tails. A fat tail means that extremely large observations are much more likely than with ‘regular statistical’ distributions like the Normal bell curve. The difference matters. Consider, for example, the distribution of cash payments described in chapter 1. If we had fitted a Normal curve (which has a thin tail) to the sample, it would have predicted that only 0.35% of payments are over 100 euro. In fact it is 10 times a much: 3.5%. The difference gets larger the further out you go: the fitted Normal distribution predicts that only 1 in every 10 billion cash payments is over 200 euro. The fitted Log-normal puts it at 0.67% of all cash payments, very close the actually observed 0.57% of the sample.

The human intuition has difficulty with such long tails and tends to underestimate the probability of extreme events which are also known as ‘black swans’. The formal definition of a fat tail says that if you go out far enough on the tail, the chance that an observation larger than x is essentially equal to the chance that it is larger than $x+s$, where s is a finite amount. The non-mathematical version of this rule is easier to grasp: “If it is bad, it is probably worse than you think.”

10 How big is that payment? The frequency distribution of payments by size

Chapter 1 analyzed the size of cash payments. We now do the same for interbank transactions over networks such as SWIFT and Fedwire. These are several orders of magnitude larger than cash transactions: the average SWIFT payment is the equivalent of 400,000 euro and the average Fedwire payment is even larger: 1,200,000 dollar. Interestingly their size follows the same type of statistical distribution as cash payments, namely the Log-normal distribution described in chapter 1. Figure 15 shows a histogram for all payments made over the SWIFT network during Oct 2010, using the logarithm of the transaction size.

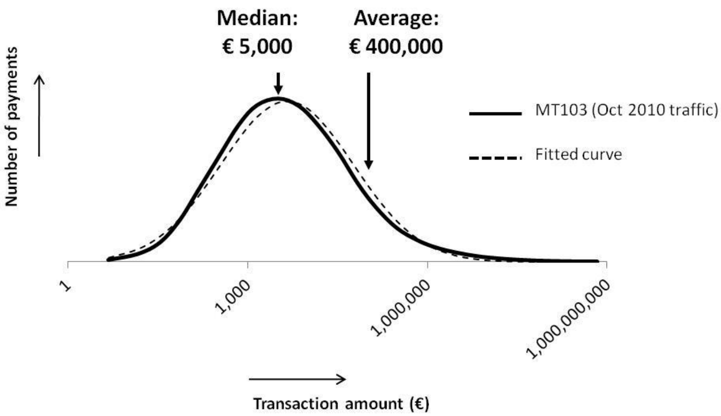


Figure 15: Distribution of SWIFT transfer instructions by amount

The figure also shows a fitted Log-normal curve. [46] The actual observed curve has a thinner left-hand tail and a thicker right-hand tail than the fitted Log-normal curve.[47]

We saw in chapter 1 that small cash payments follow this same statistical distribution, albeit with different parameters. There is evidence that the sizes of other payment mechanisms also follow a Log-normal distribution. Table 4 below gives an overview:

Table 4: frequency distribution by size of selected payment instruments[48]

Instrument	**Average value**	**Median value**	$\hat{\mu}$	$\hat{\sigma}$
Cash	25	15	2.7	1.1
Debit cards	65	43	3.8	0.9
SWIFT	400,000	5,000	8.4	2.5
T2	1,250,000	20,000	9.9	2.9
Fedwire	3,000,000	30,000	10.3	3.7

The obvious question is: why? What process would generate this apparently pervasive distribution of payment size? The honest answer is: we don't know. We know several processes that ultimately lead to a Log-normal distribution.

The central limit theorem states that the sum of a large enough number of variables, each with an identical distribution, will

[46] With Maximum Likelihood Estimate (MLE) parameters $\hat{\mu}$ =8.4 and $\hat{\sigma}$ =2.5.

[47] This is in line with the fact that the observed average (about EUR 400,000) is higher than the theoretical average which is equal to $e^{\hat{\mu}+½\hat{\sigma}^2} = 101{,}215$.

[48] Figures for Cash from Boeschoten, Fase (1989), De Grauwe, Buyst 2000) and Kippers (2004), Debit Card figures from BIS, T2 from Rosati, Secola (2005) and Fedwire from Soramäki, Bech (2006). All values in euro, except Fedwire, which are in US dollars.

eventually approach a Normal distribution. The interesting part is that the variables can have any distribution (as long as they are identical and independent of each other) yet still their sum will (eventually) follow a Normal distribution. Similarly, the product of a large number of variables, each with an identical distribution, will eventually be Log-normally distributed.

We can speculate about which multiplicative process drives payment size. It remains striking that the same distribution applies across a wide range of payment instrument and payment sizes.

Interpreting the parameters μ and σ

In the Normal distribution μ is the median (middle) observation and σ denotes confidence intervals: 2/3rds of observations fall in the interval (μ-σ, μ+σ), a range with width 2σ around the middle. For the Log-normal distribution the median is equal to e^{μ}, where *e* is Euler's constant described in chapter 6. Parameter σ still gives a sense of spread but differently. 2/3rds of observations fall between the median *multiplied* by e^{σ} and the median *divided* by e^{σ}.

For cash payments, the median is 15 euro, while $e^{\sigma} = e^{1.1} \approx 3$. So 2/3rds of observations fall into the interval from 15/3 to 15*3. This range covers a factor 9, or close to one order of magnitude (a factor 10). For the SWIFT transactions we get $e^{2.5} \approx 12$ and the interval covers a factor 144 or slightly over 2 orders of magnitude. In fact a good rule of thumb is that 75% of observations fall in an interval that covers σ orders of magnitude.

80/20 and beyond

The economist Vifredo Pareto observed that US incomes follow a Power-law with α around 1.5. The number of people earning more than 20,000 is 30 times the number earning more than 200,000 and 1000 times the number of people earning 2 million or more.[49]

The "80/20 rule" is also called the Pareto principle, after his observation that 20% of the people own 80% of the land. There is a relation with the parameter α: for α=1.5 we get "70/30": 30% of people earn 70% of income, and the top 1% earns slightly over 22%. It takes α=1.16 to get to 80/20, where the top 1% has 50%.

[49] since α=1.5, if the probability decreases by a factor 10 then the income grows by $10^{1.5}\approx 30$, and if it decreases by a factor 100 then income grows by $100^{1.5}=1000$

11 Does your amount start with a 1? Bedford's law

If numbers are more or less evenly spread over multiple orders of magnitude, Benford's law applies: the first digit of the amount is much more likely to be 1 than 9. Bedford law's predicts that for 30% of all payments the first digit of the amount is a 1, while it is a 9 for only 5% of payments.

Figure 16 below shows the distribution by leading digit for a sample of SWIFT transfers, which is remarkably close to Benford's law. Perhaps the only noteworthy deviation is the relatively high frequency of the digit 5. One explanation could be that the limit for free of charge SEPA transfers is 50,000 Euro.

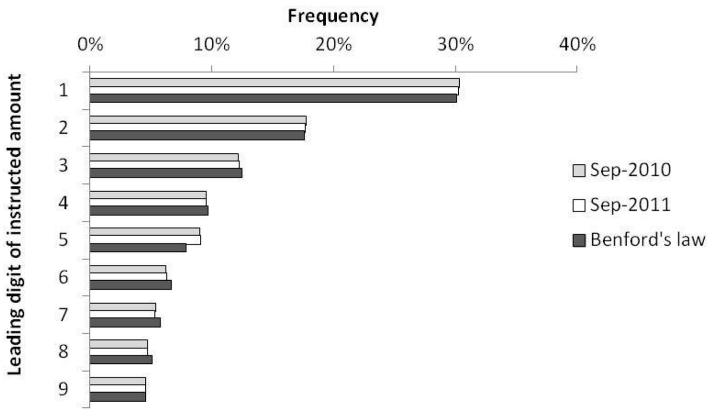
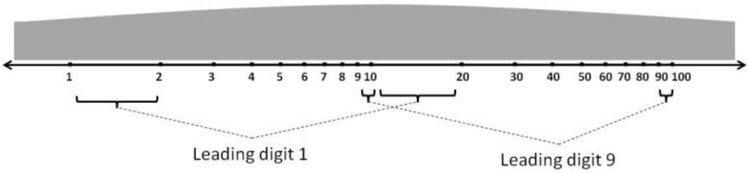


Figure 16: Benford's law compared to SWIFT transfers

Benford's law applies to any set of numbers that spans several orders of magnitude such as incomes on tax returns, annual sales of firms, and the population of cities. An interesting, and often quoted, application of Benford's law is fraud prevention, where allegedly cooked books were be spotted because fraudsters used a random generator to for each digit, putting all digits with equal frequency in the lead position.

Benford's law

To see why the digit 1 is more likely, consider the below scale: if numbers are equally likely to be between 1 and 10 as they are to be between 10 and 100, a log scale would be the appropriate one to use. On this scale, numbers starting with 1 occupy much more space than higher digits. So it seems logical that the leading digit 1 occurs most often.



The mathematics follow quite easily from this log scale: let p_n be the probability that the first digit is n, then according to Benford's law:

$$p_n = \log_{10}(n+1) - \log_{10} n = \log_{10}\left(1 + \frac{1}{n}\right).$$

This works out to about 30% for the digit 1 and only 5% for the digit 9, quite a difference.

12 What is the topology of your payment network? Node-degree distributions

Payment networks have a structure that can be analyzed and described using tools from graph theory. To do this, we take the banks to be the nodes of the network. If two banks exchange payments they share a link. This allows us to apply many insights from other networks to banking.

The number of links is also called the node-degree of a node. The statistical distribution of this node-degree reveals a lot about the nature of the network. If links are formed randomly between nodes, the node-degree distribution follows a rather “tame” distribution: most nodes will have 1 or 2 links while there may be a few with as much as 10 or 12. Analysis of real-world networks like the Internet, however, reveals that most nodes indeed have a few links, but a few nodes have 10,000 or even 1 million links. Such nodes with a high node-degree act as the hubs of the network. The node-degree in these real-world networks appears to follow a Power-law distribution.

Research has found such structures in networks as diverse as the pages of the world-wide-web, board directors of fortune 1000 companies, and the spread of AIDS. [50]

Networks with such ‘megahubs’ are formed through a process of preferential attachment of new nodes: links do not form randomly, but instead a new node is much more likely to link itself to an existing node that already has a high number of links.

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[50] Barabási (2003). For the www, the nodes are pages and the links are the hyperlinks to other pages. For company directors the directors are the nodes, 2 directors are said to be linked if they serve on the same board. For the spread of AIDS, the nodes are infected individuals who share a link if one has infected the other.

Analysis of payment networks also reveals such long tails and megahubs. For example Figure 17 shows the node-degree distribution for the banks on the SWIFT network.

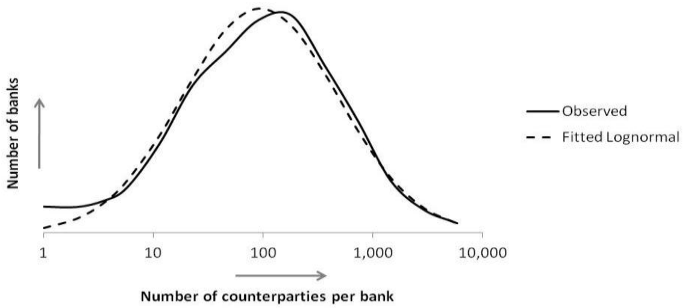


Figure 17: node-degree distribution of the SWIFT network

The actual curve is close to a normal distribution, suggesting that the underlying variable is Log-normally distributed. The dotted line plots a fitted curve.[51] Figure 18 shows the corresponding log-log frequency plot.

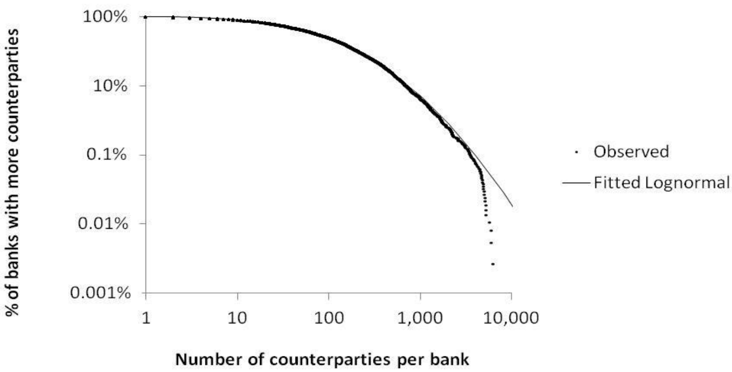


Figure 18: Log-log frequency plot of node-degree on SWIFT network

[51] With parameters $\hat{\mu}$ =4.9 and $\hat{\sigma}$ =1.7. As before these were calculated using the Maximum Likelihood Estimators.

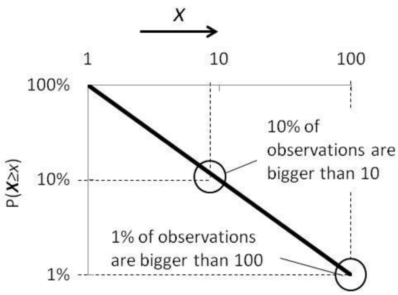
Here we find that for the upper tail the distribution falls below the Log-normal distribution. One explanation could be that the total number of Banks on the SWIFT network puts an upper limit on the number of links for any node.

Power-law distributions and log-log frequency plots

A Power-law is a type of relationship between the size events and their frequency. Consider the strength of earthquakes as measured on the Richter scale (where a force 7 quake is 10 times as strong as a force 6 event). In Southern California earthquakes exceeding force 5 on the Richter scale happen about once a year while earthquakes exceeding force 6 happen every 10 years, and earthquakes exceeding force 7 occur only once in 100 years.

Formally, a Power-law states that $P(X \geq x)=x^{-\alpha}$, where $P(X \geq x)$ denotes the probability that some variable X is bigger than a specific value x. The Power-law distribution has a key parameter α which happens to be equal to 1 in the case of earthquakes.

A key feature of Power-law distributions is that the log-log plot shows a straight line, where the slope of the line is equal to the parameter α. The plot below shows this for a Power-law distribution with α=1.



The sizes of human settlements, the intensity of wars, the size of meteorites, income and wealth, the size of files sent over the Internet and natural phenomena such as rainfall, hurricanes and earthquakes all appear to follow a Power-law.

Note that Figure 18 follows a straight line for the middle part. Several other studies of other payments networks, such as Fedwire, CHAPS and BOJ-net, have found the node-degree distribution to follow power laws. Table 5 summarizes the results.

Table 5: Node-degree distributions of selected payment networks[52]

Network	**Size (# nodes)**	**Average node-degree**	**Distribution**
SWIFT	9,000	295	*Log-normal* (μ=4.9; σ=1.7)
Fedwire	6,660	15.2	Power-law (α=1.1)
CHAPS	337	2.9	NA
BOJ-net			Power-law (α=1.3)

These findings are relevant, because scale–free networks are different from random networks in several aspects. First, the average path-length is shorter. A path is a set of (directed) links that allows one to go from one node to another. Consider the extreme case of a hub and spoke network: one megahub in the middle connected to all other nodes: here it takes at most 2 steps to go from any node to any other. In a random (Erdõs, Rényi) network the average path can be much longer. The megahubs effectively serve as conduits to shorten the paths.

Second, much analysis has been done on the vulnerability of networks to (systemic) failures. Typically, scale free networks are robust to random failures, because most nodes have few links. If nodes fail at random, the probability that a systemically important hub fails is quite low. By contrast such networks are very vulnerable to attacks that target the megahubs.

[52] Figures for Fedwire from Soramäki, Bech (2006), Chaps from Beckhr, Millard (2008), BOJ-net from Inaoka, Ninomaya (2004).

13 How big is your bank?

Since bigger banks are more likely to have a lot of interbank connections than smaller banks, it logical question arising from the previous chapter is: how are banks distributed by size? It depends of course on the definition of size, which could be assets, total revenues, number of branches, employees etc. The analysis in this chapter uses data on total assets for 28,000 banks. The largest bank had 1,5 trillion EUR, the smallest had 100,000 EUR while the average was 9 billion. A frequency plot of the logarithm of the assets yields the curve in Figure 19, together with a fitted Log-normal curve.[53]

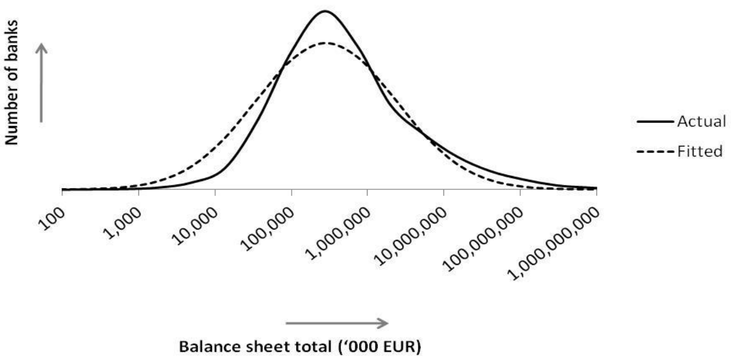


Figure 19: distribution of banks by total assets

Figure 20 shows the analysis of the right hand tail through a cumulative log-log plot. For the middle of the range the distribution appears to follow a straight line that stays above the fitted Log-normal curve. But the curve then drops off steeply for the very largest banks.

[53] The parameters are $\hat{\mu}$ =12.6 and $\hat{\sigma}$ =2.1 and were obtained using the maximum likelihood estimation method.

The same Log-normal distribution has been found for the size of (non-financial) firms.[54]

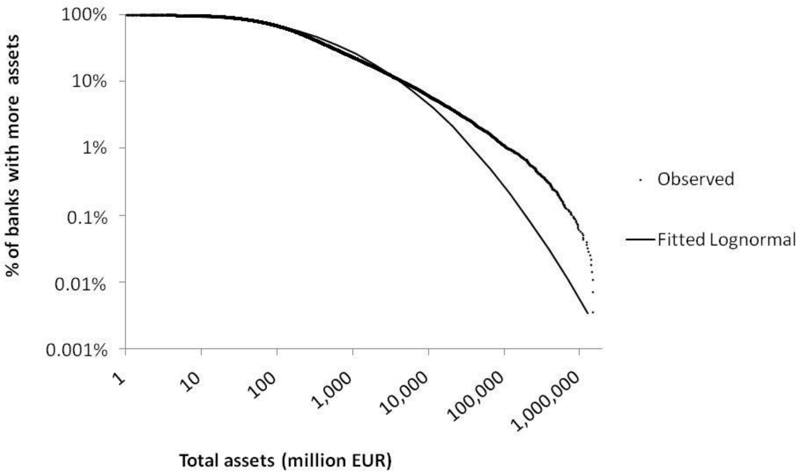


Figure 20: log-log plot of bank size by assets

[54] Stanley, Buldyrev (1995).

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